

CSci 4271W  
Development of Secure Software Systems  
Day 20: Cryptography: public key

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## Outline

- Building a secure channel
- Announcements intermission
- Public-key crypto basics
- Good technical writing (pt. 1)
- Public key encryption and signatures

## Session keys

- Don't use your long term password, etc., directly as a key
- Instead, *session key* used for just one channel
- In modern practice, usually obtained with public-key crypto
- Separate keys for encryption and MACing

## Order of operations

- Encrypt and MAC ("in parallel")
  - Safe only under extra assumptions on the MAC
- Encrypt then MAC
  - Has cleanest formal safety proof
- MAC then Encrypt
  - Preferred by FS&K for some practical reasons
  - Can also be secure

## Authenticated encryption modes

- Encrypting and MACing as separate steps is about twice as expensive as just encrypting
- "Authenticated encryption" modes do both at once
  - Newer (circa 2000) innovation, many variants
- NIST-standardized and unpatented: Galois Counter Mode (GCM)

## Ordering and message numbers

- Also don't want attacker to be able to replay or reorder messages
- Simple approach: prefix each message with counter
- Discard duplicate/out-of-order messages

## Padding

- Adjust message size to match multiple of block size
- To be reversible, must sometimes make message longer
- E.g.: for 16-byte block, append either 1, or 2 2, or 3 3 3, up to 16 "16" bytes

## Padding oracle attack

- Have to be careful that decoding of padding does not leak information
- E.g., spend same amount of time MACing and checking padding whether or not padding is right
- Remote timing attack against CBC TLS published 2013

## Don't actually reinvent the wheel

- This is all implemented carefully in OpenSSL, SSH, etc.
- Good to understand it, but rarely sensible to reimplement it
- You'll probably miss at least one of decades' worth of attacks

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## Prof. McCamant extra office hour

- Supplement for project-related demand
- Tomorrow, Friday 2-3pm
- Usual location: 4-225E Keller Hall

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## Pre-history of public-key crypto

- First invented in secret at GCHQ
- Proposed by Ralph Merkle for UC Berkeley grad. security class project
  - First attempt only barely practical
  - Professor didn't like it
- Merkle then found more sympathetic Stanford collaborators named Diffie and Hellman

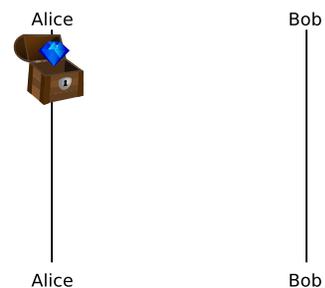
## Box and locks analogy

- Alice wants to send Bob a gift in a locked box
  - They don't share a key
  - Can't send key separately, don't trust UPS
  - Box locked by Alice can't be opened by Bob, or vice-versa

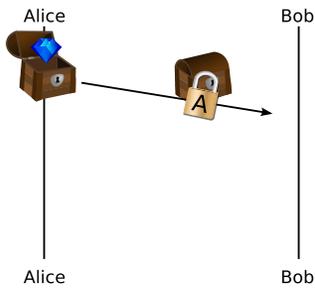
## Box and locks analogy

- Alice wants to send Bob a gift in a locked box
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- Math perspective: physical locks commute

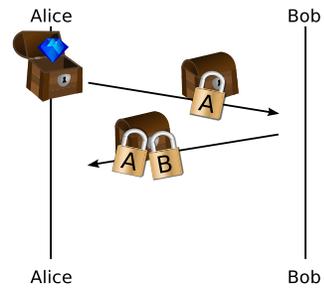
## Protocol with clip art



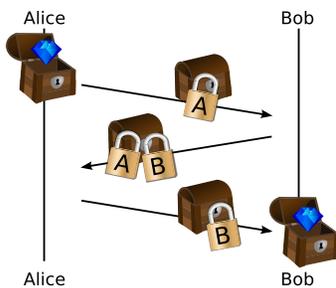
### Protocol with clip art



### Protocol with clip art



### Protocol with clip art



### Public key primitives

- Public-key encryption (generalizes block cipher)
  - Separate encryption key EK (public) and decryption key DK (secret)
- Signature scheme (generalizes MAC)
  - Separate signing key SK (secret) and verification key VK (public)

### Modular arithmetic

- Fix *modulus*  $n$ , keep only remainders mod  $n$ 
  - mod 12: clock face; mod  $2^{32}$ : unsigned int
- $+$ ,  $-$ , and  $\times$  work mostly the same
- Division? Multiplicative inverse by extended GCD
- Exponentiation: efficient by square and multiply

### Generators and discrete log

- Modulo a prime  $p$ , non-zero values and  $\times$  have a nice ("group") structure
- $g$  is a *generator* if  $g^0, g, g^2, g^3, \dots$  cover all elements
- Easy to compute  $x \mapsto g^x$
- Inverse, *discrete logarithm*, hard for large  $p$

### Diffie-Hellman key exchange

- Goal: anonymous key exchange
- Public parameters  $p, g$ ; Alice and Bob have resp. secrets  $a, b$
- Alice  $\rightarrow$  Bob:  $A = g^a \pmod{p}$
- Bob  $\rightarrow$  Alice:  $B = g^b \pmod{p}$
- Alice computes  $B^a = g^{ba} = k$
- Bob computes  $A^b = g^{ab} = k$

### Relationship to a hard problem

- We're not sure discrete log is hard (likely not even NP-complete), but it's been unsolved for a long time
- If discrete log is easy (e.g., in P), DH is insecure
- Converse might not be true: DH might have other problems

## Categorizing assumptions

- Math assumptions unavoidable, but can categorize
- E.g., build more complex scheme, shows it's "as secure" as DH because it has the same underlying assumption
- Commonly "decisional" (DDH) and "computational" (CDH) variants

## Key size, elliptic curves

- Need key sizes ~10 times larger than security level
  - Attacks shown up to about 768 bits
- Elliptic curves: objects from higher math with analogous group structure
  - (Only tenuously connected to ellipses)
- Elliptic curve algorithms have smaller keys, about  $2\times$  security level

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## Writing in CS versus other writing

- Key goal is accurately conveying precise technical information
- More important: careful use of terminology, structured organization
- Less important: writer's personality, persuasion, appeals to emotion

## Still important: concise expression

- Don't use long words or complicated expressions when simpler ones would convey the same meaning. Negative examples:
  - necessitate
  - utilize
  - due to the fact that
- Beneficial for both clarity and style

## Know your audience: terminology

- When technical terminology makes your point clearly, use it
- Provide definitions if a concept might be new to many readers
  - Be careful to provide the right information in the definition
  - Define at the first instead of a later use
- But, avoid introducing too many new terms
  - Keep the same term when referring to the same concept

## Precise explanations

- Don't say "we" do something when it's the computer that does it
  - And avoid passive constructions
- Don't anthropomorphize (computers don't "know")
- Use singular by default so plural provides a distinction:
  - The students take tests
  - + Each student takes a test
  - + Each student takes two tests

## Provide structure

- Use plenty of sections and sub-sections
- It's OK to have some redundancy in previewing structure
- Limit each paragraph to one concept, and not too long
  - Start with a clear topic sentence
- Split long, complex sentences into separate ones

## Know your audience: Project 0.5

- For projects in this course, assume your audience is another student who already understands general course concepts
  - Up to the current point in the course
  - I.e., don't need to define "buffer overflow" from scratch
- But you need to explain specifics of a vulnerable program
  - Make clear what part of the program you're referring to
  - Explain all the specific details of a vulnerability

## Inclusive language

- Avoid words and grammar that implies relevant people are male
- My opinion: avoid using he/him pronouns for unknown people
- Some possible alternatives
  - "he/she"
  - Alternating genders
  - Rewrite to plural and use "they" (may be less clear)
  - Singular "they" (least traditional, but spreading)

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## General description

- Public-key encryption (generalizes block cipher)
  - Separate encryption key EK (public) and decryption key DK (secret)
- Signature scheme (generalizes MAC)
  - Separate signing key SK (secret) and verification key VK (public)

## RSA setup

- Choose  $n = pq$ , product of two large primes, as modulus
- $n$  is public, but  $p$  and  $q$  are secret
- Compute encryption and decryption exponents  $e$  and  $d$  such that

$$M^{ed} = M \pmod{n}$$

## RSA encryption

- Public key is  $(n, e)$
- Encryption of  $M$  is  $C = M^e \pmod{n}$
- Private key is  $(n, d)$
- Decryption of  $C$  is  $C^d = M^{ed} = M \pmod{n}$

## RSA signature

- Signing key is  $(n, d)$
- Signature of  $M$  is  $S = M^d \pmod{n}$
- Verification key is  $(n, e)$
- Check signature by  $S^e = M^{de} = M \pmod{n}$
- Note: symmetry is a nice feature of RSA, not shared by other systems

## RSA and factoring

- We're not sure factoring is hard (likely not even NP-complete), but it's been unsolved for a long time
- If factoring is easy (e.g., in P), RSA is insecure
- Converse might not be true: RSA might have other problems

## Homomorphism

- Multiply RSA ciphertexts  $\Rightarrow$  multiply plaintexts
- This *homomorphism* is useful for some interesting applications
- Even more powerful: fully homomorphic encryption (e.g., both  $+$  and  $\times$ )
  - First demonstrated in 2009; still challenging

## Problems with vanilla RSA

- Homomorphism leads to chosen-ciphertext attacks
- If message and  $e$  are both small compared to  $n$ , can compute  $M^{1/e}$  over the integers
- Many more complex attacks too

## Hybrid encryption

- Public-key operations are slow
- In practice, use them just to set up symmetric session keys
- + Only pay RSA costs at setup time
- Breaks at either level are fatal

## Padding, try #1

- Need to expand message (e.g., AES key) size to match modulus
- PKCS#1 v. 1.5 scheme: prepend 00 01 FF FF .. FF
- Surprising discovery (Bleichenbacher'98): allows adaptive chosen ciphertext attacks on SSL
  - Variants recurred later (c.f. "ROBOT" 2018)

## Modern "padding"

- Much more complicated encoding schemes using hashing, random salts, Feistel-like structures, etc.
- Common examples: OAEP for encryption, PSS for signing
- Progress driven largely by improvement in random oracle proofs

## Simpler padding alternative

- "Key encapsulation mechanism" (KEM)
- For common case of public-key crypto used for symmetric-key setup
  - Also applies to DH
- Choose RSA message  $r$  at random mod  $n$ , symmetric key is  $H(r)$
- Hard to retrofit, RSA-KEM insecure if  $e$  and  $r$  reused with different  $n$

## Post-quantum cryptography

- One thing quantum computers would be good for is breaking crypto
- Square root speedup of general search
  - Countermeasure: double symmetric security level
- Factoring and discrete log become poly-time
  - DH, RSA, DSA, elliptic curves totally broken
  - Totally new primitives needed (lattices, etc.)
- Not a problem yet, but getting ready

## Box and locks revisited

- Alice and Bob's box scheme fails if an intermediary can set up two sets of boxes
  - Middleperson (man-in-the-middle) attack
- Real world analogue: challenges of protocol design and public key distribution