

CSci 4271W  
Development of Secure Software Systems  
Day 21: Cryptography part 3

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## Outline

- Block ciphers modes of operation
- Hash functions and MACs
- Announcements intermission
- Building a secure channel
- Public-key crypto basics
- Public key encryption and signatures

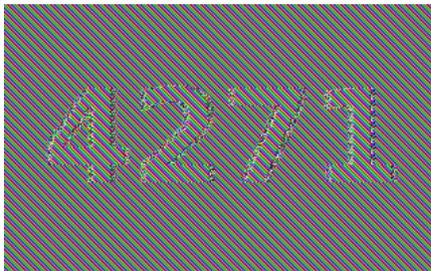
## Modes of operation

- How to build a cipher for arbitrary-length data from a block cipher
- Many approaches considered
  - For some reason, most have three-letter acronyms
- More recently: properties susceptible to relative proof

## ECB

- Electronic CodeBook
- Split into blocks, apply cipher to each one individually
- Leaks equalities between plaintext blocks
- Almost never suitable for general use

## Do not use ECB



## CBC

- Cipher Block Chaining
- $C_i = E_K(P_i \oplus C_{i-1})$
- Long-time most popular approach, starting to decline
- Plaintext changes propagate forever, ciphertext changes only one block

## CBC: getting an IV

- $C_0$  is called the initialization vector (IV)
  - Must be known for decryption
- IV should be random-looking
  - To prevent first-block equalities from leaking (lesser version of ECB problem)
- Common approaches
  - Generate at random
  - Encrypt a nonce

## Stream modes: OFB, CTR

- Output FeedBack: produce keystream by repeatedly encrypting the IV
  - Danger: collisions lead to repeated keystream
- Counter: produce from encryptions of an incrementing value
  - Recently becoming more popular: allows parallelization and random access

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## Ideal model

- Ideal crypto hash function: pseudorandom function
  - Arbitrary input, fixed-size output
- Simplest kind of elf in box, theoretically very convenient
- But large gap with real systems: better practice is to target particular properties

## Kinds of attacks

- Pre-image, "inversion": given  $y$ , find  $x$  such that  $H(x) = y$
- Second preimage, targeted collision: given  $x$ ,  $H(x)$ , find  $x' \neq x$  such that  $H(x') = H(x)$
- (Free) collision: find  $x_1, x_2$  such that  $H(x_1) = H(x_2)$

## Birthday paradox and attack

- There are almost certainly two people in this class with the same birthday
- $n$  people have  $\binom{n}{2} = \Theta(n^2)$  pairs
- So only about  $\sqrt{n}$  expected for collision
- "Birthday attack" finds collisions in any function

## Security levels

- For function with  $k$ -bit output:
- Preimage and second preimage should have complexity  $2^k$
- Collision has complexity  $2^{k/2}$
- Conservative: use hash function twice as big as block cipher key
  - Though if you're paranoid, cipher blocks can repeat too

## Non-cryptographic hash functions

- The ones you probably use for hash tables
- CRCs, checksums
- Output too small, but also not resistant to attack
- E.g., CRC is linear and algebraically nice

## Short hash function history

- On the way out: MD5 (128 bit)
  - Flaws known, collision-finding now routine
- SHA(-0): first from NIST/NSA, quickly withdrawn
  - Likely flaw discovered 3 years later
- SHA-1: fixed SHA-0, 160-bit output.
- $2^{60}$  collision attack described in 2013
  - First public collision found (using 6.5 kCPU yr) in 2017

## Length extension problem

- MD5, SHA1, etc., computed left to right over blocks
- Can sometimes compute  $H(a \parallel b)$  in terms of  $H(a)$ 
  - $\parallel$  means bit string concatenation
- Makes many PRF-style constructions insecure

## SHA-2 and SHA-3

- SHA-2: evolutionary, larger, improvement of SHA-1
  - Exists as SHA-{224, 256, 384, 512}
  - But still has length-extension problem
- SHA-3: chosen recently in open competition like AES
  - Formerly known as Keccak, official standard Aug. 2015
  - New design, fixes length extension
  - Adoption has been gradual

## MAC: basic idea

- Message authentication code: similar to hash function, but with a key
- Adversary without key cannot forge MACs
- Strong definition: adversary cannot forge anything, even given chosen-message MACs on other messages

## CBC-MAC construction

- Same process as CBC encryption, but:
  - Start with IV of 0
  - Return only the last ciphertext block
- Both these conditions needed for security
- For fixed-length messages (only), as secure as the block cipher

## HMAC construction

- $H(K \parallel M)$ : insecure due to length extension
  - Still not recommended:  $H(M \parallel K)$ ,  $H(K \parallel M \parallel K)$
- HMAC:  $H(K \oplus a \parallel H(K \oplus b \parallel M))$
- Standard  $a = 0x5c^*$ ,  $b = 0x36^*$
- Probably the most widely used MAC

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## Project 1 submission 2 extended

- New deadline: next Wednesday 4/13, 11:59pm
  - One-time extension will be until Friday night
- My comments will be finished by Friday
  - Visible gradually as they are posted

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## Session keys

- Don't use your long term password, etc., directly as a key
- Instead, *session key* used for just one channel
- In modern practice, usually obtained with public-key crypto
- Separate keys for encryption and MACing

## Order of operations

- Encrypt and MAC (“in parallel”)
  - Safe only under extra assumptions on the MAC
- Encrypt then MAC
  - Has cleanest formal safety proof
- MAC then Encrypt
  - Preferred by FS&K for some practical reasons
  - Can also be secure

## Authenticated encryption modes

- Encrypting and MACing as separate steps is about twice as expensive as just encrypting
- “Authenticated encryption” modes do both at once
  - Newer (circa 2000) innovation, many variants
- NIST-standardized and unpatented: Galois Counter Mode (GCM)

## Ordering and message numbers

- Also don’t want attacker to be able to replay or reorder messages
- Simple approach: prefix each message with counter
- Discard duplicate/out-of-order messages

## Padding

- Adjust message size to match multiple of block size
- To be reversible, must sometimes make message longer
- E.g.: for 16-byte block, append either 1, or 2 2, or 3 3 3, up to 16 “16” bytes

## Padding oracle attack

- Have to be careful that decoding of padding does not leak information
- E.g., spend same amount of time MACing and checking padding whether or not padding is right
- Remote timing attack against CBC TLS published 2013

## Don’t actually reinvent the wheel

- This is all implemented carefully in OpenSSL, SSH, etc.
- Good to understand it, but rarely sensible to reimplement it
- You’ll probably miss at least one of decades’ worth of attacks

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## Pre-history of public-key crypto

- First invented in secret at GCHQ
- Proposed by Ralph Merkle for UC Berkeley grad. security class project
  - First attempt only barely practical
  - Professor didn’t like it
- Merkle then found more sympathetic Stanford collaborators named Diffie and Hellman

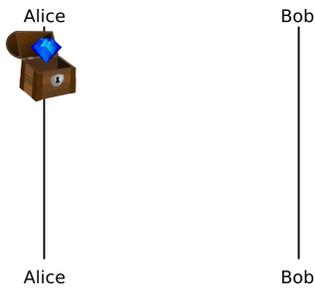
## Box and locks analogy

- Alice wants to send Bob a gift in a locked box
  - They don't share a key
  - Can't send key separately, don't trust UPS
  - Box locked by Alice can't be opened by Bob, or vice-versa

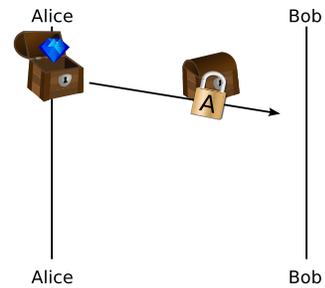
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- Alice wants to send Bob a gift in a locked box
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  - Box locked by Alice can't be opened by Bob, or vice-versa
- Math perspective: physical locks commute

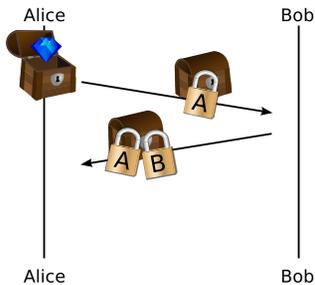
## Protocol with clip art



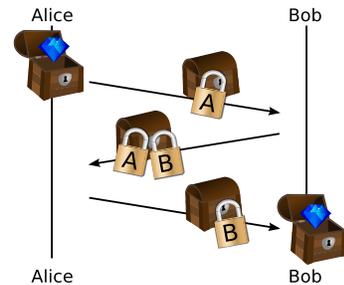
## Protocol with clip art



## Protocol with clip art



## Protocol with clip art



## Public key primitives

- Public-key encryption (generalizes block cipher)
  - Separate encryption key EK (public) and decryption key DK (secret)
- Signature scheme (generalizes MAC)
  - Separate signing key SK (secret) and verification key VK (public)

## Modular arithmetic

- Fix *modulus*  $n$ , keep only remainders mod  $n$ 
  - mod 12: clock face; mod  $2^{32}$ : unsigned int
- $+$ ,  $-$ , and  $\times$  work mostly the same
- Division? Multiplicative inverse by extended GCD
- Exponentiation: efficient by square and multiply

## Generators and discrete log

- Modulo a prime  $p$ , non-zero values and  $\times$  have a nice ("group") structure
- $g$  is a *generator* if  $g^0, g, g^2, g^3, \dots$  cover all elements
- Easy to compute  $x \mapsto g^x$
- Inverse, *discrete logarithm*, hard for large  $p$

## Diffie-Hellman key exchange

- Goal: anonymous key exchange
- Public parameters  $p, g$ ; Alice and Bob have resp. secrets  $a, b$
- Alice  $\rightarrow$  Bob:  $A = g^a \pmod{p}$
- Bob  $\rightarrow$  Alice:  $B = g^b \pmod{p}$
- Alice computes  $B^a = g^{ba} = k$
- Bob computes  $A^b = g^{ab} = k$

## Relationship to a hard problem

- We're not sure discrete log is hard (likely not even NP-complete), but it's been unsolved for a long time
- If discrete log is easy (e.g., in P), DH is insecure
- Converse might not be true: DH might have other problems

## Categorizing assumptions

- Math assumptions unavoidable, but can categorize
- E.g., build more complex scheme, shows it's "as secure" as DH because it has the same underlying assumption
- Commonly "decisional" (DDH) and "computational" (CDH) variants

## Key size, elliptic curves

- Need key sizes  $\sim 10$  times larger than security level
  - Attacks shown up to about 768 bits
- Elliptic curves: objects from higher math with analogous group structure
  - (Only tenuously connected to ellipses)
- Elliptic curve algorithms have smaller keys, about  $2\times$  security level

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## General description

- Public-key encryption (generalizes block cipher)
  - Separate encryption key EK (public) and decryption key DK (secret)
- Signature scheme (generalizes MAC)
  - Separate signing key SK (secret) and verification key VK (public)

## RSA setup

- Choose  $n = pq$ , product of two large primes, as modulus
- $n$  is public, but  $p$  and  $q$  are secret
- Compute encryption and decryption exponents  $e$  and  $d$  such that

$$M^{ed} = M \pmod{n}$$

## RSA encryption

- Public key is  $(n, e)$
- Encryption of  $M$  is  $C = M^e \pmod{n}$
- Private key is  $(n, d)$
- Decryption of  $C$  is  $C^d = M^{ed} = M \pmod{n}$

## RSA signature

- Signing key is  $(n, d)$
- Signature of  $M$  is  $S = M^d \pmod{n}$
- Verification key is  $(n, e)$
- Check signature by  $S^e = M^{de} = M \pmod{n}$
- Note: symmetry is a nice feature of RSA, not shared by other systems

## RSA and factoring

- We're not sure factoring is hard (likely not even NP-complete), but it's been unsolved for a long time
- If factoring is easy (e.g., in P), RSA is insecure
- Converse might not be true: RSA might have other problems

## Homomorphism

- Multiply RSA ciphertexts  $\Rightarrow$  multiply plaintexts
- This *homomorphism* is useful for some interesting applications
- Even more powerful: fully homomorphic encryption (e.g., both  $+$  and  $\times$ )
  - First demonstrated in 2009; still very inefficient

## Problems with vanilla RSA

- Homomorphism leads to chosen-ciphertext attacks
- If message and  $e$  are both small compared to  $n$ , can compute  $M^{1/e}$  over the integers
- Many more complex attacks too

## Hybrid encryption

- Public-key operations are slow
- In practice, use them just to set up symmetric session keys
- + Only pay RSA costs at setup time
- Breaks at either level are fatal

## Padding, try #1

- Need to expand message (e.g., AES key) size to match modulus
- PKCS#1 v. 1.5 scheme: prepend 00 01 FF FF .. FF
- Surprising discovery (Bleichenbacher'98): allows adaptive chosen ciphertext attacks on SSL
  - Variants recurred later (c.f. "ROBOT" 2018)

## Modern "padding"

- Much more complicated encoding schemes using hashing, random salts, Feistel-like structures, etc.
- Common examples: OAEP for encryption, PSS for signing
- Progress driven largely by improvement in random oracle proofs

### Simpler padding alternative

- "Key encapsulation mechanism" (KEM)
- For common case of public-key crypto used for symmetric-key setup
  - Also applies to DH
- Choose RSA message  $r$  at random mod  $n$ , symmetric key is  $H(r)$
- Hard to retrofit, RSA-KEM insecure if  $e$  and  $r$  reused with different  $n$

### Post-quantum cryptography

- One thing quantum computers would be good for is breaking crypto
- Square root speedup of general search
  - Countermeasure: double symmetric security level
- Factoring and discrete log become poly-time
  - DH, RSA, DSA, elliptic curves totally broken
  - Totally new primitives needed (lattices, etc.)
- Not a problem yet, but getting ready

### Box and locks revisited

- Alice and Bob's box scheme fails if an intermediary can set up two sets of boxes
  - Middleperson (man-in-the-middle) attack
- Real world analogue: challenges of protocol design and public key distribution