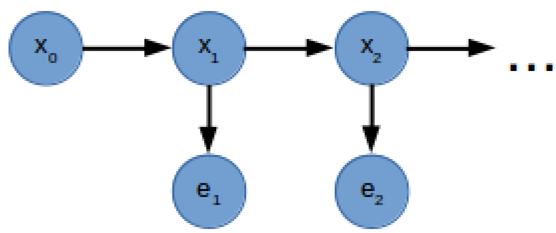
5512, Spring-2019
ASSIGNMENT 3:
Assigned: 03/11/19 Due: 03/31/19 at 11:55 PM (submit via Canvas, you may scan or take a picture of your paper answers) <u>Submit only pdf or txt files (for non-code part), separate submission for code files</u>
<u>Show as much work as possible for all problems!</u>

Problems 1 to 4 will use the following Bayesian network (specifically, a Hidden Markov Model): (Here X's represent whether the water table is low/medium/high and E's represent whether or not your basement gets flooded.)

$P(x_0)$	
low	1/3
med	1/3
high	1/3

$P(x_{t+1} \mid x_t)$		to x_{t+1}: low	med	high
from x_t	low	0.6	0.35	0.05
	med	0.2	0.6	0.2
	high	0	0.5	0.5



$P(e_t \mid x_t)$		e_t seen: flooded	not flooded
x_t	low	0	1
	med	0.05	0.95
	high	0.4	0.6

Problem 1. (15 points)
Give the filtering probabilities for the water table's three values in the following cases:
(1) $P(x_1 \mid e_1=$"not flooded"$)$
(2) $P(x_2 \mid e_1=$"not flooded", $e_2=$"not flooded"$)$
(3) $P(x_3 \mid e_1=$"not flooded", $e_2=$"not flooded", $e_3=$"flooded"$)$

Problem 2. (15 points)
Given the same sequence of evidence as problem 1, find the smoothed estimates for x_1, x_2 and x_3. (In other words, the evidence is still: $e_1=$"not flooded", $e_2=$"not flooded", $e_3=$"flooded")

Then plot the probabilities both filtering and smoothed estimates on the same graph. (Note: you will need two points/lines for a single probability, so overall you should have four lines.)

Problem 3. (20 points)
Assume we are using the same HMM as problems 1 & 2, but we have difference evidence:
$e_1=$"flooded", $e_2=$"not flooded", $e_3=$"not flooded", $e_4=$"flooded", $e_5=$"not flooded"

What is the most likely sequence of water table levels for these five days?

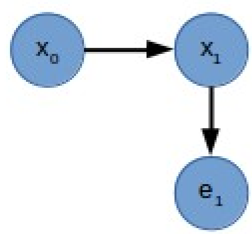
If you found that day 6 was "flooded" (i.e. $e_6=$"flooded"), what is the most likely sequence now?

Problem 4. (25 points)
Use particle filtering to estimate:
P(x_{10} | e_1="not flooded", e_2="not flooded", e_3="flooded", e_4="flooded", e_5="not flooded", e_6="not flooded", e_7="not flooded", e_8="not flooded", e_9="flooded", e_{10}="not flooded")

(i.e. the days "flooded" are 3,4 and 9. The rest are "not flooded".) Give the number of particles used in your sampling, along with the probability for the water table values.

Problem 5. (20 points)
Assume we are using the Frisbee example from class, where: $P(x_0) = N(0,1)$ and $P(e_t|x_t) = N(x_t, 0.75)$. How accurate do you need to be so that after 10 throws, the variance is not more than 10.

Problem 6. (15 points)
Suppose you have the simple HMM 3-node Bayes net shown below. Assume that $P(x_0)$ is uniformly distributed between [-1, 1]. The transition probability, $P(x_{t+1} \mid x_t)$, is $N(x_t,1)$. The evidence probability is: $P(e_t|x_t) = N(x_t, 0.5)$



Use this description to approximate the distribution for $P(x_1 \mid e_1=0)$ and plot this distribution on a graph. Extend this network to 5 nodes to estimate the distribution for $P(x_2 \mid e_1=0, e_2=0)$ and plot this distribution on a graph. As $t \rightarrow \infty$, describe what happens to the distribution of the filtering probabilities (assuming the evidence always says zero).