2nd Midterm Exam

Thursday April 6
75 minutes – Open book and notes

1. *10 points*
   Convert these English sentences to predicate calculus, using the following predicates: $Person(x), School(y), Class(z), Athlete(v), In(x, z), Faster(x, w)$

   1. John is faster than any other person in his class.
   2. Every school has a class where at least one person in the class is an athlete.

2. *10 points*
   Specify if each of the following expressions represents correctly the corresponding English statement. If not explain why not and correct it.

   1. There is a cat in each house.
      $\forall x \forall y \ [House(x) \wedge Cat(y) \rightarrow In(y, x)]$
   2. Every cat owner loves all animals.
      $\exists x \exists y \ [Owns(x, y) \wedge Cat(y)] \wedge \forall z \ [Animal(z) \wedge Loves(x, z)]$

3. *10 points*
   Write the following sentences in English, making clear what are the differences between them:

   1. $\forall x \exists y \ Class(x) \Rightarrow Student(y) \wedge In(y, x)$
   2. $\exists y \forall x \ Student(y) \wedge Class(x) \Rightarrow In(y, x)$
   3. $\exists x \exists y \ Class(x) \wedge Student(y) \wedge In(y, x)$
   4. $\forall x \forall y \ Class(x) \wedge Student(y) \Rightarrow In(y, x)$

4. *15 points*
   Convert to following expressions to CNF

   1. $(B \vee (A \wedge C)) \Rightarrow (B \vee \neg A)$
   2. $\forall p \ [[Pet(p) \wedge \exists c \ [Owner(c, p) \vee Feeds(c, p)]] \Rightarrow Happy(p)]$
   3. $\forall x \exists y \forall z \ [P(x, y, z) \Rightarrow [\exists u \ Q(x, u)]]$

5. *10 points*

   1. Convert the following sentences in propositional logic to CNF:

      1. $\neg A \Rightarrow B \vee C$
      2. $A \Rightarrow B$
      3. $\neg(\neg B \Rightarrow D)$

   2. Prove "$C \wedge \neg D$" using resolution with refutation. Show the steps in the resolution proof.

6. *25 points*
   You are given the following knowledge about some blocks:

   1. A is on B, B on C. A is green, C is blue.
   2. If something is green it is not blue.

   Prove using resolution with refutation that "there is a green object on an object that is not green." [Hint: Make sure you work on the proof until you find a contraddiction].

7. *20 points*
   Answer these questions explaining your reasoning briefly but precisely.

   1. What is a Horn clause? why are Horn clauses important?
   2. Is Modus Ponens in propositional calculus complete? What does completeness mean?
   3. Entailment in propositional calculus is decidable, but in predicate calculus it is only semidecidable. What does it mean? Be precise.
   4. Suppose you use resolution with refutation to prove that $alpha \models \beta$. Does is mean that β is valid? if not, why not?

8. Extra Credit *10 points*
   Represent the sentence 'All germans speak the same language" using the following predicates: $German(x)$ x is a german; $Speaks(x, l)$ x speaks language l.

   The sentence can be written in logic in different ways. Can you think of more than one way of writing it in logic? Write at least two different logical representations for it.

   $\boxed{\text{YOU REACHED THE END OF THE EXAM}}$