CSci 2033 Practice Exercises Set #9 Feb. 28, 2018

1 Let A be an $n \times n$ invertible matrix. Prove that if u, v, w are 3 linearly independent vectors in $\mathbb{R}^n$ then Au, Av, Aw are also linearly independent.

2 Find the inverse of the matrix $A = \begin{bmatrix} -1 & -1 \\ 1 & 2 \end{bmatrix}$.

3 A linear mapping T from $\mathbb{R}^2$ to $\mathbb{R}^3$ is represented by a matrix A ('standard matrix'). What size is this matrix? Determine A if we know that

$$T\left(\begin{bmatrix} -1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix} \quad \text{and} \quad T\left(\begin{bmatrix} -1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix}$$

[Hint: If A is the sought matrix the above conditions can be written as $A[u_1, u_2] = [v_1, v_2]$. You can now use the inverse, so $A = ...$]