CSci 2033, S'18	**Homework # 1**	**Due Date: Feb 7, 2018**

1. Find the elementary row operation that transforms the frist matrix into the second and then the inverse row operation that transforms the second matrix into the first:

$$\begin{bmatrix} 2 & 1 & 4 & 0 \\ 0 & 1 & 2 & -3 \\ 0 & 3 & 10 & -7 \end{bmatrix} \quad \begin{bmatrix} 2 & 1 & 4 & 0 \\ 0 & 1 & 2 & -3 \\ 0 & 0 & 4 & 2 \end{bmatrix}$$

2. The parabola $y = a + bx + cx^2$ goes through the points $(x, y) = (1, 4)$, $(x, y) = (2, 8)$ and $(x, y) = (3, 14)$. Formulate a linear system of equations for a, b, c and find its solution with Gaussian elimination.

3. Find solutions (if any) of the following system of equations [Explain your steps and conclusions]:

$$\begin{aligned} x_1 - 5x_2 + 4x_3 &= -3 \\ 2x_1 - 7x_2 + 3x_3 &= -2 \\ -2x_1 + x_2 + 7x_3 &= -1 \end{aligned}$$

4. Consider the following system of linear equations

$$\begin{aligned} x_1 - 2x_2 - 2x_3 &= 2 \\ x_1 + 2x_2 + x_3 &= 3 \\ 3x_1 + 2x_2 \qquad &= 8 \end{aligned}$$

   a) Express this system in an augmented matrix form
   b) Transform the system into *standard row echelon* form (Show a step-by-step execution)
   c) Transform the system into *reduced row echelon* form (Show a step-by-step execution)
   d) What is the solution set of this system? Is the solution unique?

5. Same question as in the previous one but the linear system is modified by changing the last equation into $-2x_1 + x_2 + 7x_3 = -6$

6. Convert the following augmented matrices into reduced echelon form, then find the solution sets in each case, identifying the free variables if any: (Use Matlab for verification)

$$\begin{bmatrix} 3 & -3 & 6 & 0 \\ 1 & 2 & -1 & 2 \\ 0 & -3 & 3 & -2 \end{bmatrix} \quad \begin{bmatrix} 5 & 4 & 2 & 6 \\ 1 & -1 & 4 & 0 \\ 1 & 2 & 4 & 2 \end{bmatrix} \quad \begin{bmatrix} 5 & 4 & 1 & 0 \\ 1 & -1 & 2 & 0 \\ 1 & 2 & -1 & 2 \end{bmatrix}$$

7. Fertilizers are mixtures containing specified amounts of 3 elements: nitrogen (N), phosphorus (P), and potassium (K). A garden store carries three blends F1 , F2 , F3 that have the following compositions:

	N	P	K
F1	30	10	10
F2	20	20	30
F3	10	20	20

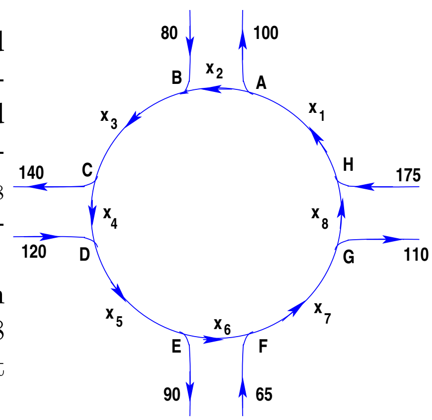
For example, F1 has 30% N, 10% P, 10% K (and 50% other elements). For those familiar with farming or gardening this is written as an N-P-K ratio of 30-10-10. What matters is the amount of each of these components relative to the others (Thus, 15-5-10 is *essentially* the same as 30-10-20).

(a) A customer has a special need for a fertilizer with proportions of N-P-K equal to 20-15-15. Find how the store should mix F1 , F2 , F3 to produce a mixture with composition required by the customer. Formulate this problem as a system of linear equations and solve it using matlab ('backslash').

(b) Assume that a customer has a special need for a fertilizer with the composition N-P-K equal to 25-5-15. Is there a solution? Is it always possible to blend the 3 fertilizers to reach *any* desired composition? Discuss.

8. (It will be helpful to first look at Example 5 of Section 1.3) Let $A = \begin{bmatrix} 2 & 0 & 6 \\ -1 & 8 & -3 \\ 1 & -2 & 3 \end{bmatrix}$, let $b = \begin{bmatrix} 10 \\ 3 \\ 7 \end{bmatrix}$, and let W be the set of all linear combinations of the columns of A.
(a) is b in W? (b) Is the second column of A is in W?

9. The figure on the right represents the traffic entering and leaving a roundabout road junction. Such roundabouts ensure a continuous and smooth flow at road junctions and are very popular in Europe. The numbers on each road entering/leaving the roundabout and the variables $x_1, \cdots, x_8$ indicate the number of vechicle per hour on each of the related sections.



(a) By stating a 'law of conservation of vehicles' at each of the 8 points A, B, ..., H, construct a linear system of 8 equations satisfied by the eight unknowns $x_1, \cdots .x_8$. What is the general solution of the system?
[Hint: Once you write the system you can use matlab's RREF to answer the question]

(b) What is the minimum flow possible over x_1?

(c) Find a solution of the system that gives $x_4 = 40$. Are there any solutions for which x_4 is equal to zero? Explain why or why not.
(d) Find a solution of the system for which x_6 is the double of x_4.

10. The following matlab commands will produce 3 figures (run it on your machine):

```
XY = [1 3 2 1; 1 2 4 1];
plot(XY(1,:),XY(2,:));
hold
XY = 0.7*XY;
plot(XY(1,:),XY(2,:));
XY = 0.7*XY;
plot(XY(1,:),XY(2,:));
```

(a) Explain the commands used to produce these figures (what is XY, what does 'hold' do, what happens at the command $XY = 0.7 * XY$, etc.).
(b) Next, find out how to have plots with a different color for each of the 3 figure (blue, red, green). Generate a new plot with these colors. [for (b) Provide a print-out of your answer and the corresponding figure with 3 colors]