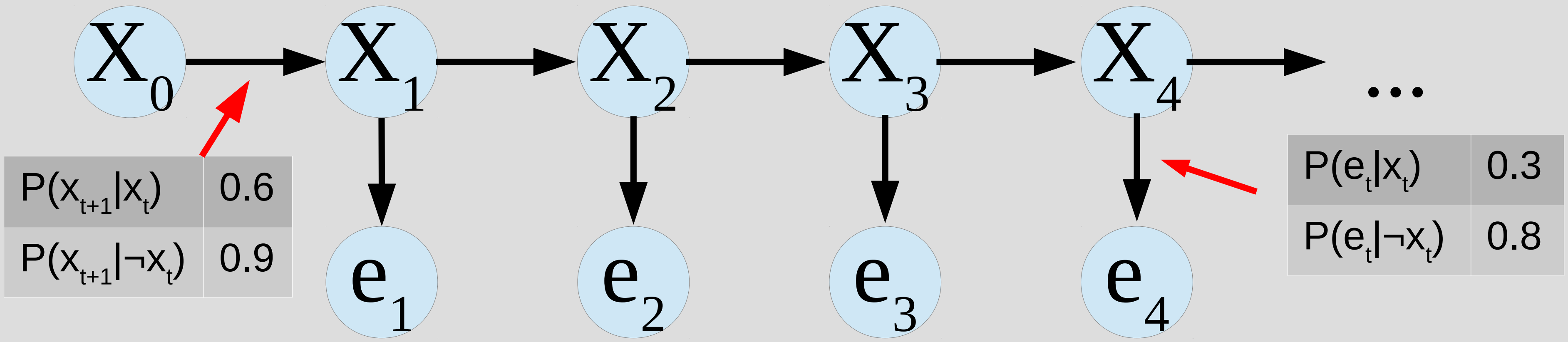
Particle Filter (Ch. 15)

WELL, WHEN WE OBSERVE THEM, THEY BECOME AMBER *PARTICLES* OF GRAIN.

Hidden Markov Model

To deal with information over time, we used a hidden Markov model:



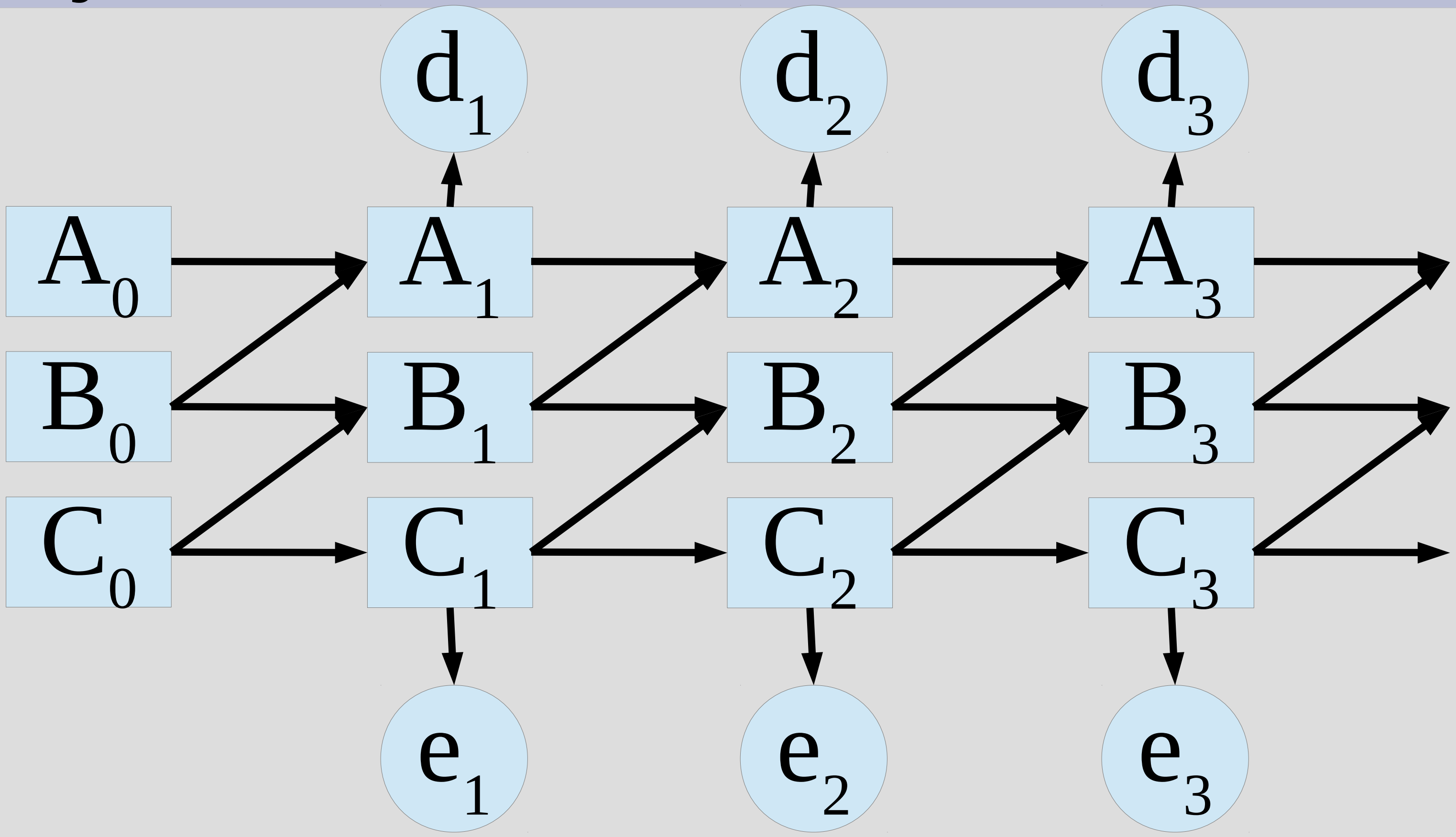
$P(x_{t+1} \mid x_t)$	0.6
$P(x_{t+1} \mid \neg x_t)$	0.9

$P(e_t \mid x_t)$	0.3
$P(e_t \mid \neg x_t)$	0.8

Often, we want more than a single variable and/or evidence for our problem
(also works well for continuous variables)

Dynamic Bayesian Network

Key: ● =evidence, ▭ =non-evidence



This more general representation is called a dynamic Bayesian network

Hidden Markov Model

We could always just cluster all evidence and all non-evidence to fit the HMM

However, this would lead to an exponential amount of calculations/table size
(as the cluster would have to track every combination of variables)

Rather, it is often better to relax our HMM assumptions and expand the network

Dynamic Bayesian Network

Unfortunately, it is harder to compute a "filtered" message in this new network

We could still follow the same process:

1. Use t_0 to compute t_1, add evidence at t_1
2. Use t_1 to compute t_2, add evidence at t_2
3. (continue)

(Similar to our "forward message" in HMMs)

Dynamic Bayesian Network

The process is actually very similar to variable elimination (with factors)

You have a factor for each variable and combine them to get the next step, then you sum out the previous step

Even with this “efficient” approach, it is still $O(d^{n+k})$, where d=domain size (2 if T/F), k=num parents, n=num var

Particle Filtering

If our network is large, finding the exact probabilities is infeasible

Instead, we will use something similar to likelihood weighting called particle filtering

This will estimate the filtered probability (i.e. $P(x_t|e_{1:t})$) using the previous estimate (i.e. $P(x_{t-1}|e_{1:t-1})$)... and then repeating

Particle Filtering

Particle filtering algorithm:

- Sample to initialize t=0 based on $P(x_0)$ with N sample “particles”
- Loop until you reach t you want:
  - (1) Sample to apply transition from t-1: each particle samples to decide where go
  - (2) Weight samples based on evidence: Weight of particle in state x is $P(e|x)$
  - (3) Resample N particles based on weights: P(particle in x) = sum w in x / total sum w

Particle Filtering

Particle filtering algorithm:

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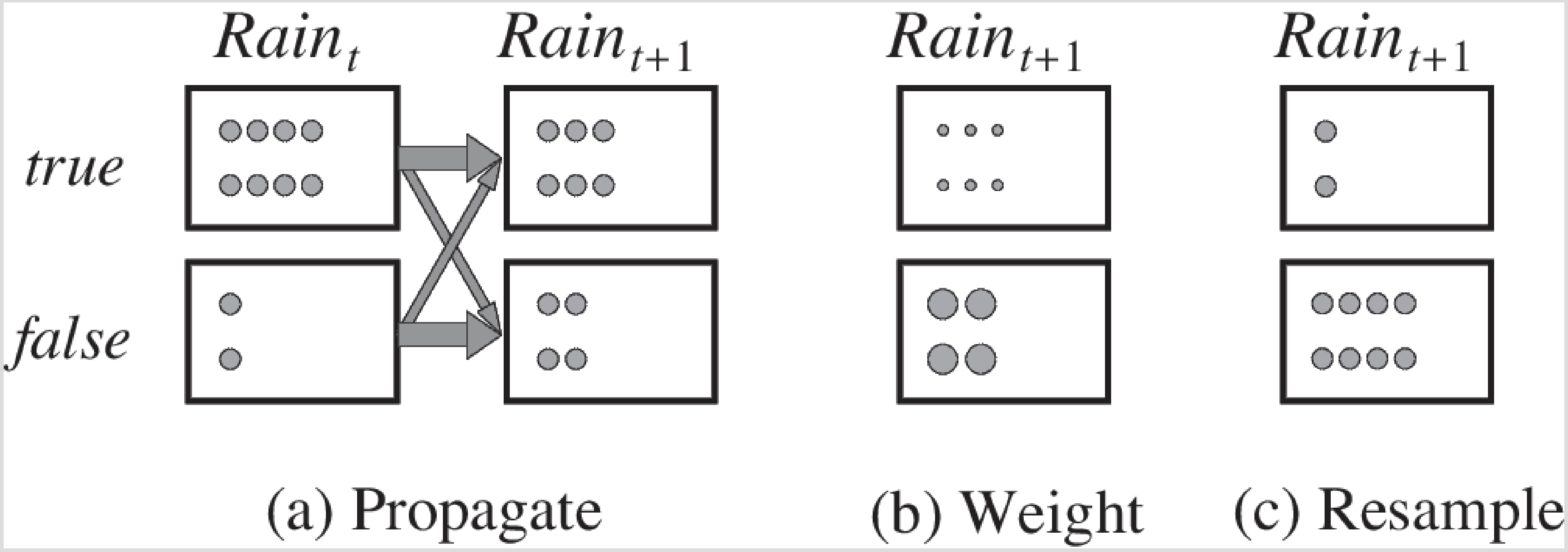
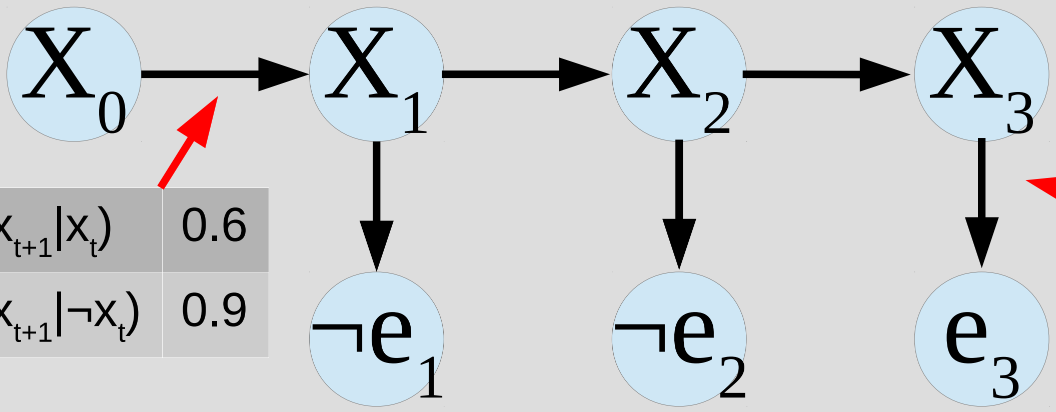


Particle Filtering

Although the algorithm is *<u>supposed</u>* to be run in a more complex network... lets start small

$P(x_0)$	0.5



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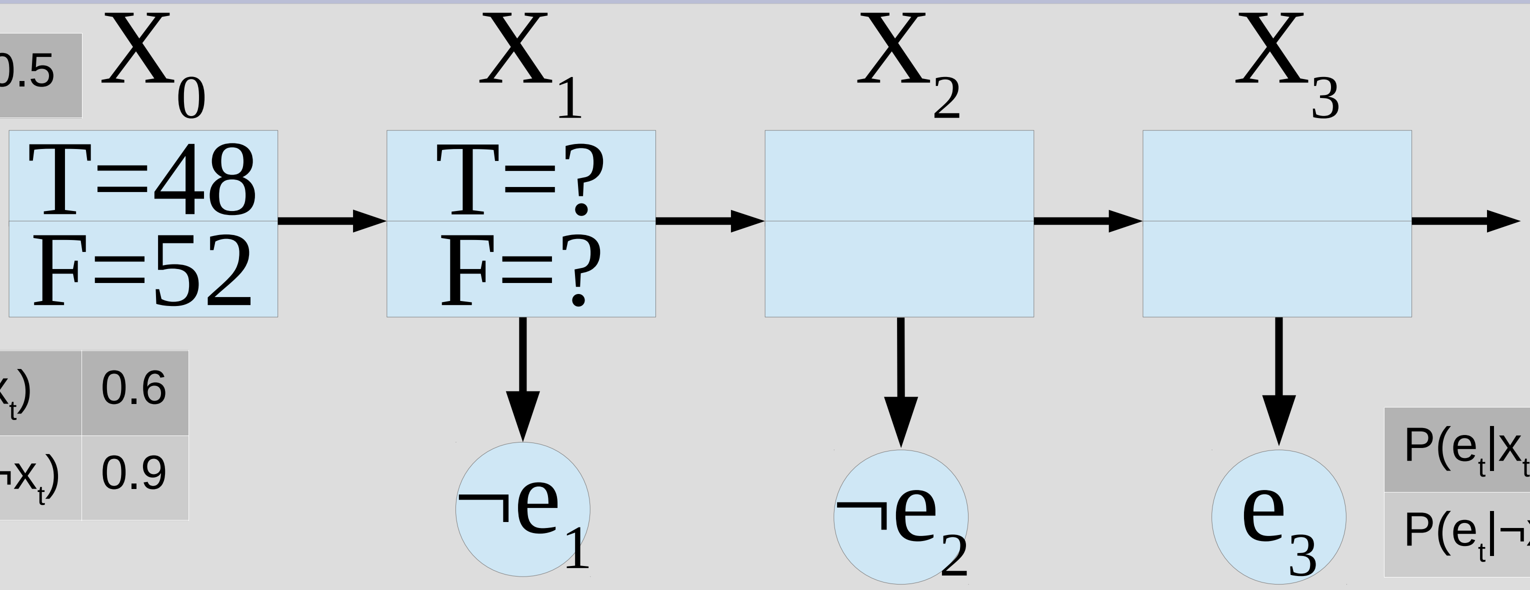
Let’s do N=100 particles

First we sample randomly to assign all 100 particles T/F in X_0

Particle Filtering

$P(x_0)$	0.5



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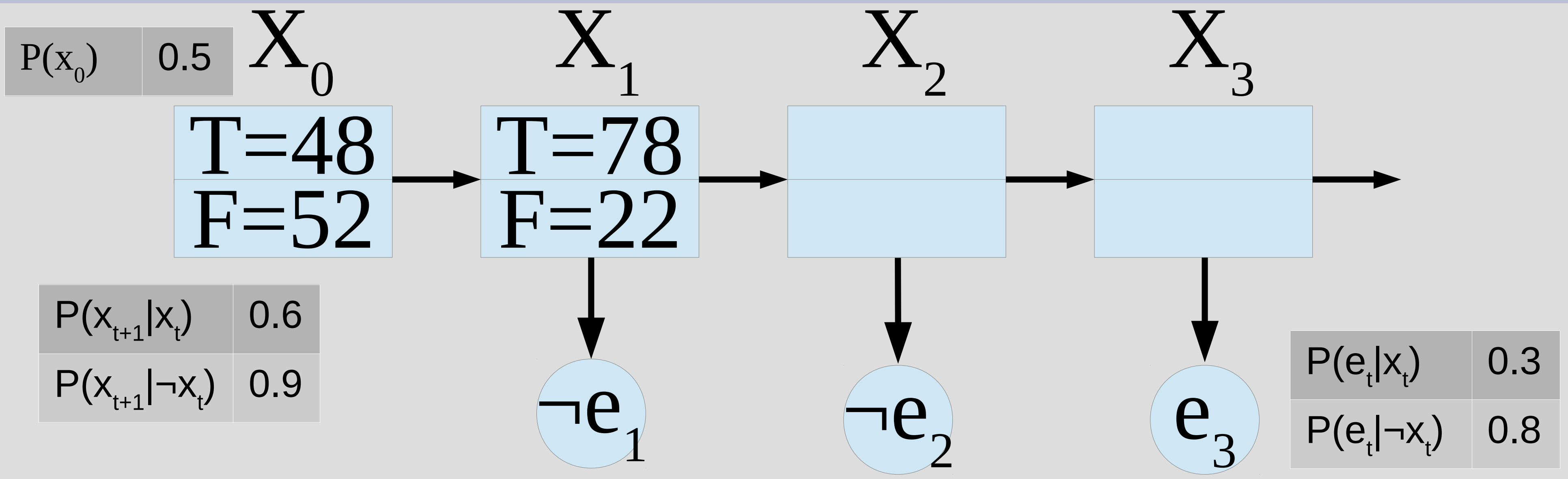
For each particle that is T is X_0:

60% chance to be T in X_1, 40% F in X_1

For each particle that is F is X_0:

90% chance to be T in X_1, 10% F in X_1

Particle Filtering



$P(x_0)$	0.5

$P(x_{t+1}\|x_t)$	0.6
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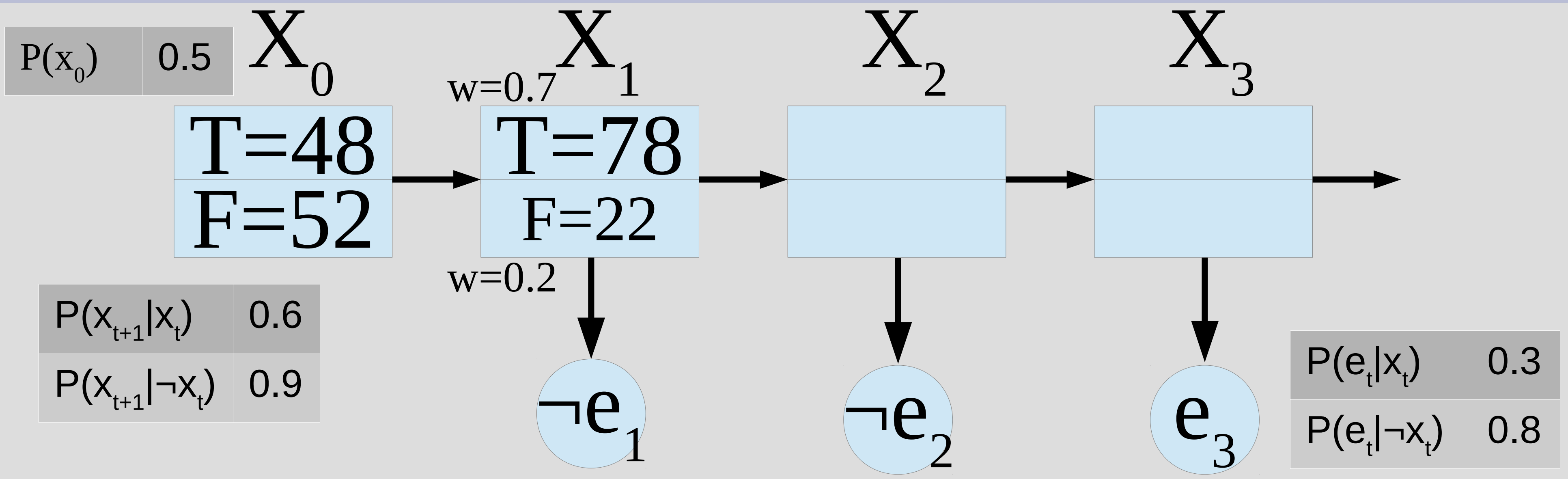
$P(e_t\|x_t)$	0.3
$P(e_t\|\neg x_t)$	0.8

Then apply evidence weight, since $\neg e_1$:

T in X_1 weighted as 0.7

F in X_1 weighted as 0.2

Particle Filtering



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Resample X1 based on weight & counts:

Total weight = 78*0.7 + 22*0.2 = 59

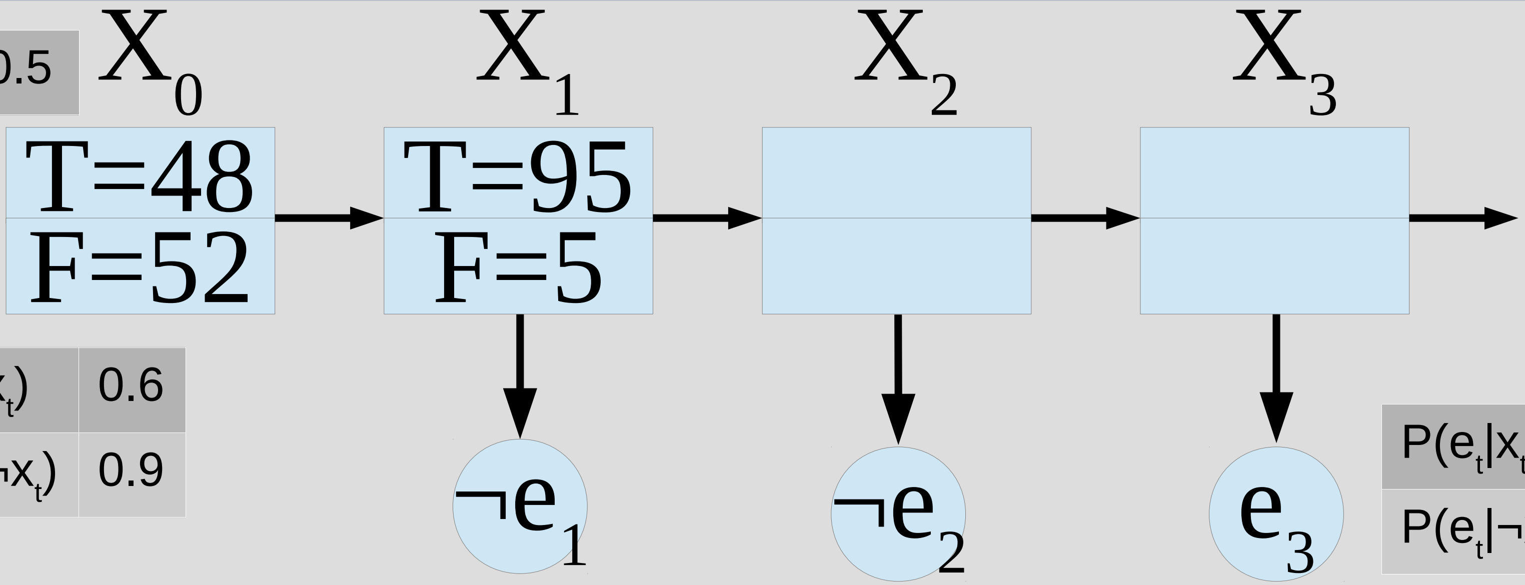
Weight in T samples = 78*0.7 = 54.6

Resample as T = 54.6/59 = 92.54%

(100 samples still)

Particle Filtering

$P(x_0)$	0.5



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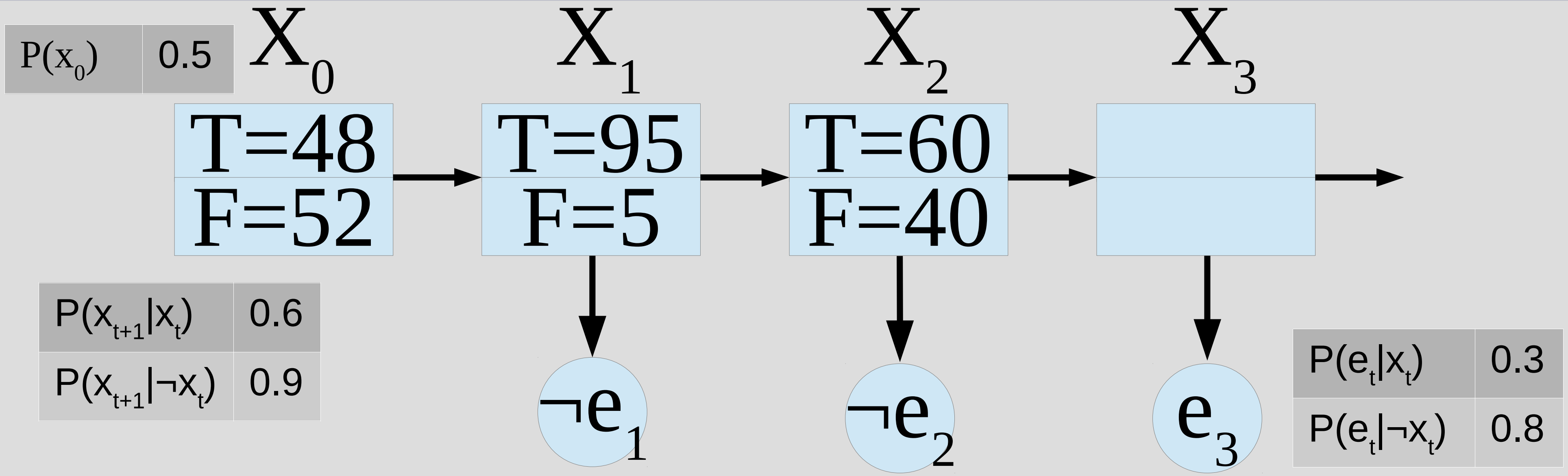
Start process again... first transition

For each particle:

X_1 T: 60% chance to be T in X_2, 40% F in X_2

X_1 F: 90% chance to be T in X_2, 10% F in X_2

Particle Filtering



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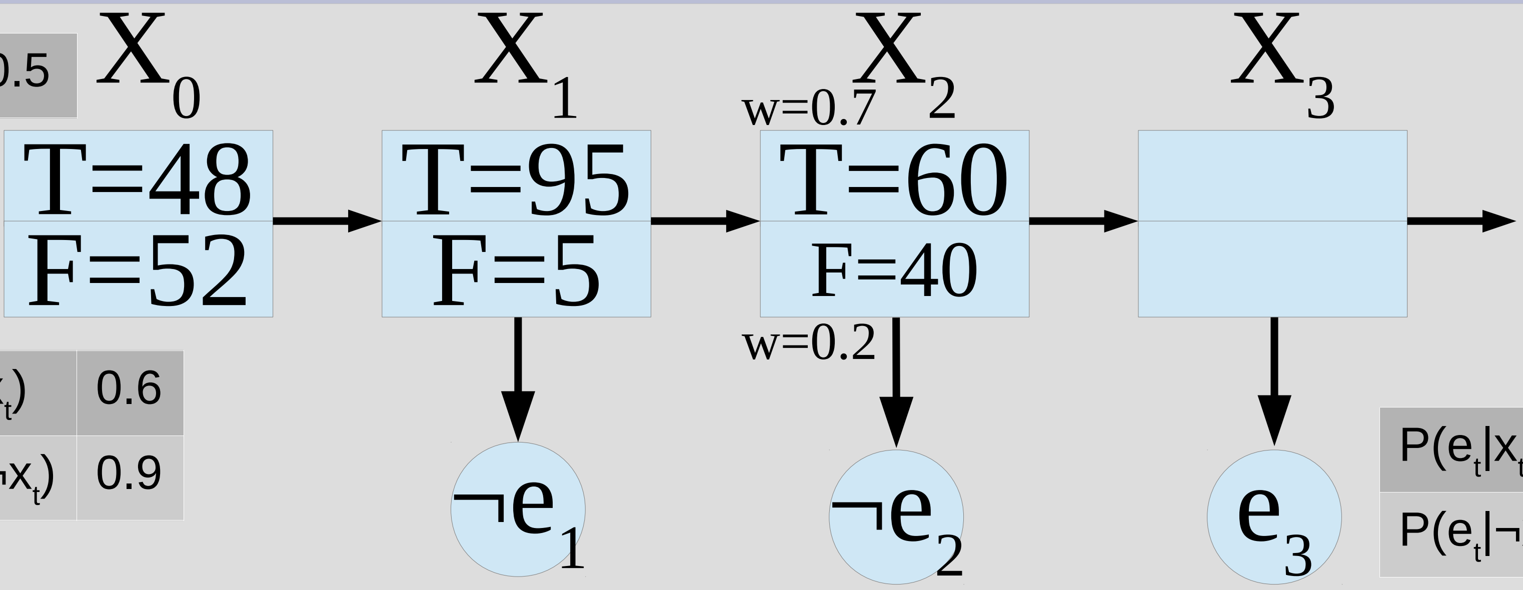
Weight evidence (same evidence as last time, so same weight):

T has $w=P(\neg e_2|x_2) = 0.7$

F has $w=P(\neg e_2|\neg x_2) = 0.2$

Particle Filtering

$P(x_0)$	0.5



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Resample:

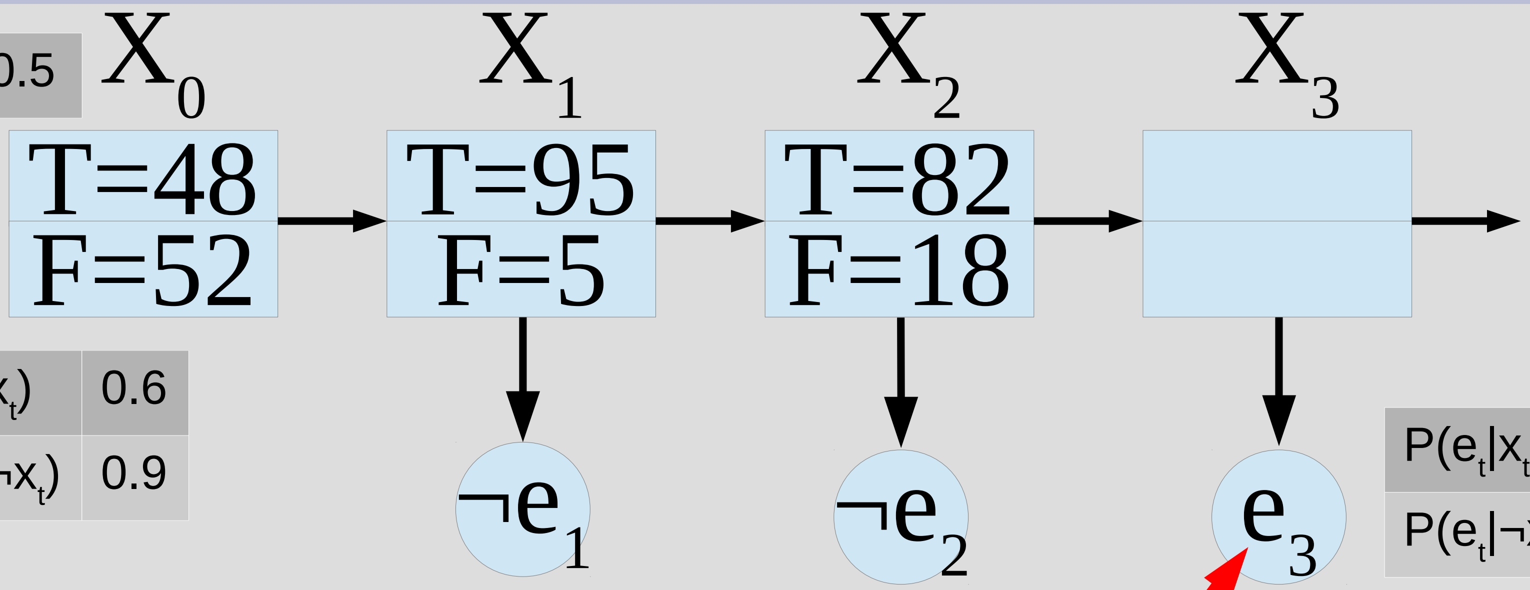
Total weight = 60*0.7 + 40*0.2 = 50

T weight = 60*0.7 = 42

P(sample T in X_2) = 42/50 = 0.84

Particle Filtering

P(x$_0$)	0.5



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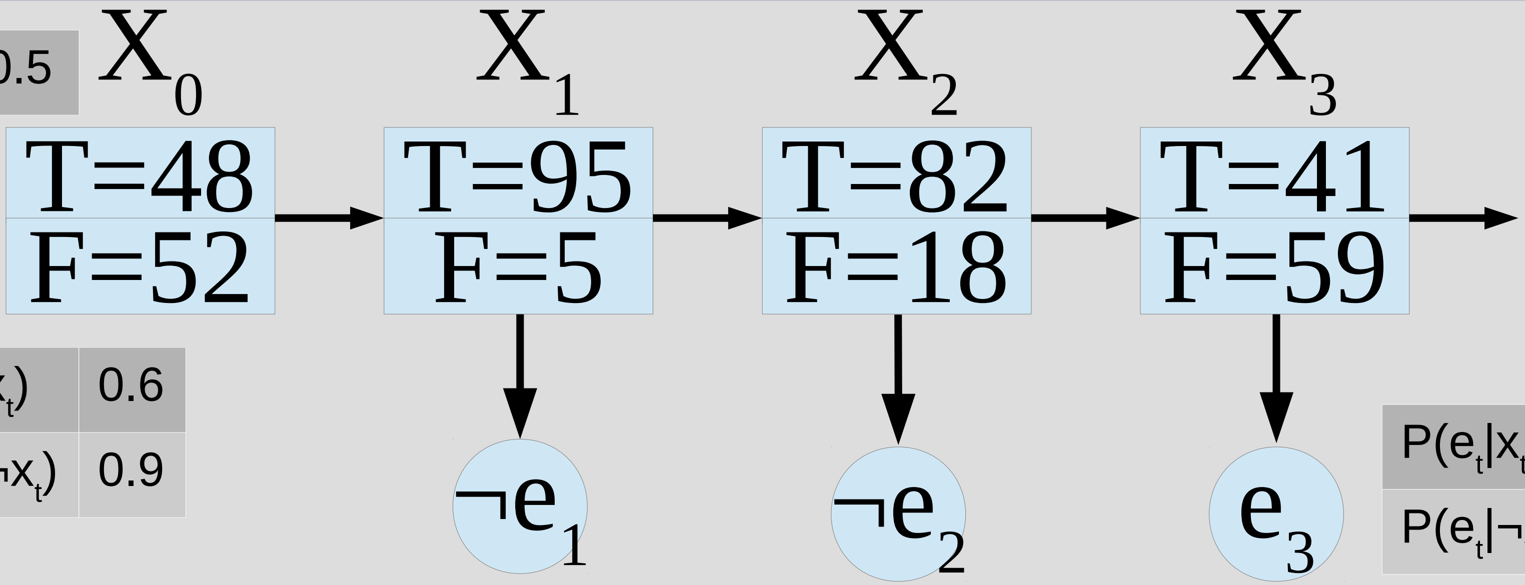
You do X_3!

evidence positive this time

(Rather than "sampling" just round to nearest if you want to check your work with here)

Particle Filtering

$P(x_0)$	0.5



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You should get:

(1) 65/35

(2) w for T = 0.3, W for F = 0.8

(3) 41/59

Particle Filtering

Why does it work?

Each step computes the next “forward” message in filtering, so we can use induction

If one forward message is done right, they should all be approximately correct

(Base case is trivial as $P(x_0)$ is directly sampled, so should be approximate correct)

Particle Filtering

We compute the probabilities as:

$$P(x_t|e_{1:t}) = \frac{\text{Number of true resamples}}{\text{Total number of samples}} = \frac{N(x_t|e_{1:t})}{N}$$

(above is our inductive hypothesis)

$$P(x_{t+1}|e_{1:t+1}) = \underbrace{P(e_{t+1}|x_{t+1})}_{\text{Step (2)}} \sum_{x_t} \underbrace{P(x_{t+1}|x_t) \cdot N(x_t|e_{1:t})}_{\text{Step (1)}}$$

$$\approx P(e_{t+1}|x_{t+1}) \sum_{x_t} P(x_{t+1}|x_t) \cdot (N \cdot P(x_t|e_{1:t}))$$

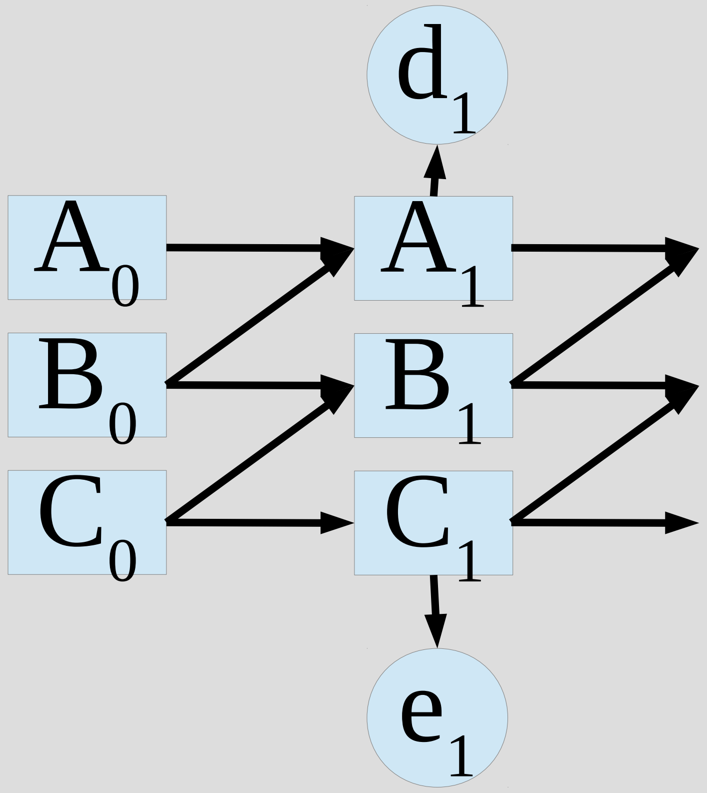
$$= N \cdot P(e_{t+1}|x_{t+1}) \sum_{x_t} P(x_{t+1}|x_t) \cdot (P(x_t|e_{1:t}))$$

Step (3) should look a lot like normalize

$$= \underbrace{\alpha}_{\text{Step (3)}} \cdot P(e_{t+1}|x_{t+1}) \sum_{x_t} P(x_{t+1}|x_t) \cdot (P(x_t|e_{1:t}))$$

Real World Complications

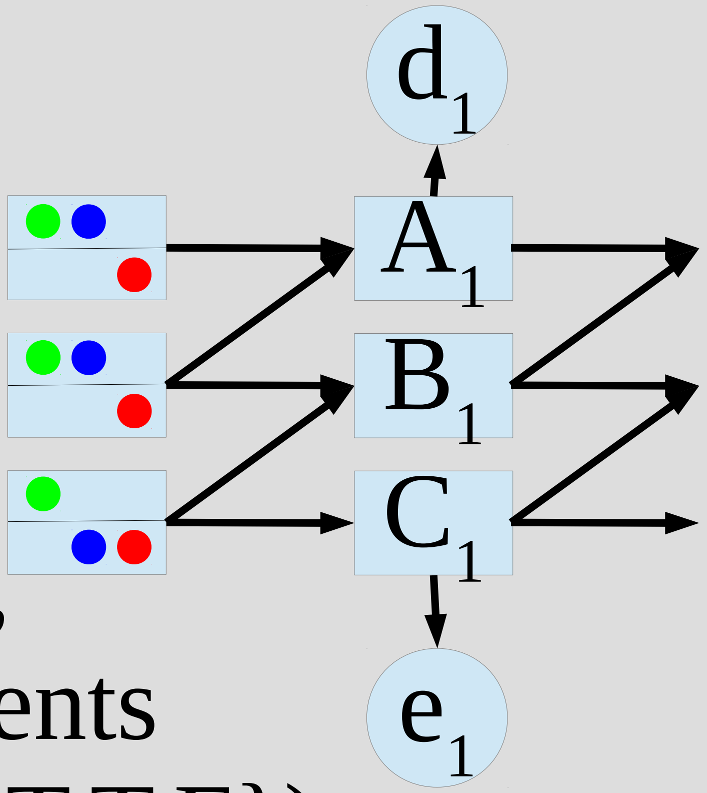
If we had a dynamic Bayes net (below), what do we need to change about particle filtering?


$$\begin{tikzcd}
 & d_1 & \\
A_0 \arrow[r] & A_1 \arrow[u] \arrow[r] & {} \\
B_0 \arrow[ur] \arrow[r] & B_1 \arrow[ur] \arrow[r] & {} \\
C_0 \arrow[ur] \arrow[r] & C_1 \arrow[ur] \arrow[r] \arrow[d] & {} \\
 & e_1 &
\end{tikzcd}$$

Real World Complications

If we had a dynamic Bayes net (below), what do we need to change about particle filtering?

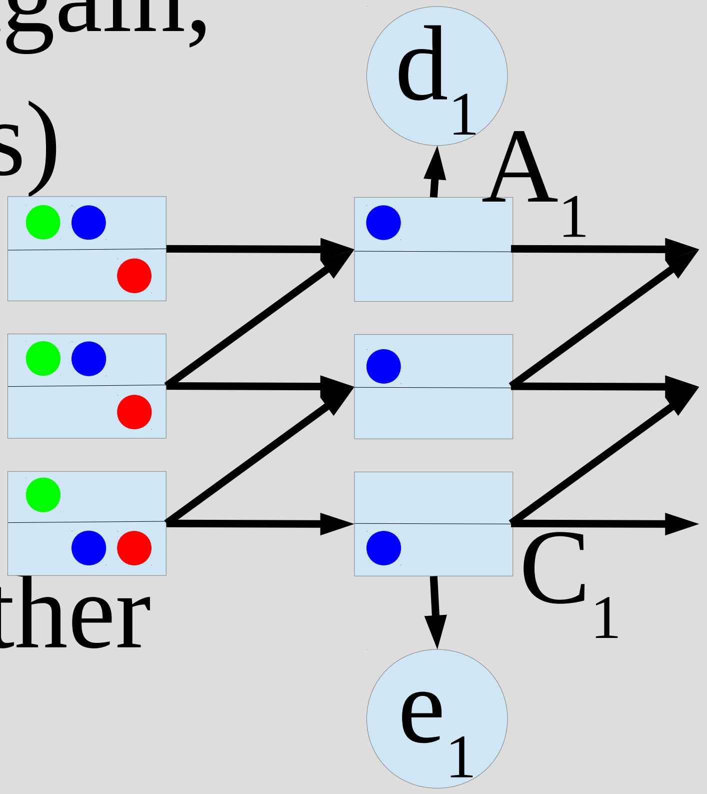
For multiple variables, a particle should represent a value in each variable



So if A,B,C are T/F variables, each color in the DBN represents a single particle (e.g. blue = {T,T,F})

Real World Complications

Thus, when finding where "blue particle" should go, you probabilistically determine a position for A_1, B_1 and C_1 (again, the particle spans all variables)



The weighting is similar to likelihood, where you just multiply all weights together (e.g. for blue particle this is: $P(e_1|\neg c_1) \cdot P(d_1|a_1)$)

Real World Complications

Biggest real world simplifications?

Real World Complications

Biggest real world simplifications?

The sensors are only considered to be "uncertain", but quite often they fail

Temporarily failures (i.e. incorrect sensor readings for a few steps) can be handled by ensuring the transition is high enough

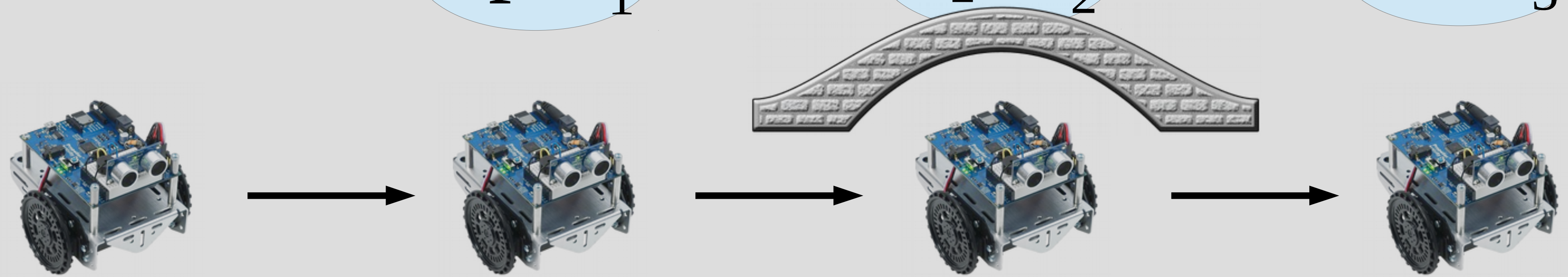
Assume 0 is a sensor failure

(i.e. P(reading = 0 | reading = valid) = 0.01)

Real World Complications

This can handle cases where there is a brief moment of failure:

$$\begin{tikzcd}
 & GPS_1 & \xcancel{GPS_2} & GPS_3 \\
Position_0 \arrow[r] & Position_1 \arrow[u] \arrow[r] & Position_2 \arrow[u] \arrow[r] & Position_3 \arrow[u] \\
Speed_0 \arrow[r] \arrow[ur] & Speed_1 \arrow[r] \arrow[ur] \arrow[d] & Speed_2 \arrow[r] \arrow[ur] \arrow[d] & Speed_3 \arrow[d] \\
 & Spin_1 & Spin_2 & Spin_3
\end{tikzcd}$$



Real World Complications

To handle cases where the sensor completely fails, you can add another variable

This new variable should have a small chance of going “false” and when false, it will always stay there and give bad readings

You can then ask the network which variable is more likely to be true, and judge off of that

Real World Complications

$$\begin{tikzcd}
\text{SensOK}_0 \arrow[r] & \text{SensOK}_1 \arrow[r] \arrow[d] & \text{SensOK}_2 \arrow[r] \arrow[d] & \text{SensOK}_3 \arrow[d] \\
 & \text{GPS}_1 & \cancel{\text{GPS}_2} & \cancel{\text{GPS}_3} \\
\text{Position}_0 \arrow[r] & \text{Position}_1 \arrow[r] \arrow[u] & \text{Position}_2 \arrow[r] \arrow[u] & \text{Position}_3 \arrow[u] \\
\text{Speed}_0 \arrow[r] \arrow[ur] & \text{Speed}_1 \arrow[r] \arrow[ur] \arrow[d] & \text{Speed}_2 \arrow[r] \arrow[ur] \arrow[d] & \text{Speed}_3 \arrow[d] \\
 & \text{Spin}_1 & \text{Spin}_2 & \text{Spin}_3
\end{tikzcd}$$