QR algorithm

1. Basic idea $A = QR$, $A := RQ$ repeat

2. Shifts of origin $A - s I = QR$ ...
   -> to make the process more efficient

3. Deflation
   Up to this point Cost is $O(n^4)$

4. Pre-transform A into Hessenberg form
   total cost → $O(n^3)$

5. Implementation details:
   Bulge chasing etc.

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  x x
  x x x
    x x x
      x x x
        x x
```

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F: mxn n ≤ m

$$\begin{vmatrix} I & F \\ F^T & 0 \end{vmatrix} = \begin{vmatrix} I & 0 \\ F^T & I \end{vmatrix} \begin{vmatrix} I & F \\ 0 & S \end{vmatrix} = \begin{vmatrix} I & 0 \\ F^T & I \end{vmatrix} \begin{vmatrix} I & 0 \\ 0 & S \end{vmatrix} \begin{vmatrix} I & F \\ 0 & I \end{vmatrix}$$

$$L \qquad D \quad \rightarrow F^T F + S = 0 \rightarrow S = -F^T F$$

inertia = [n , 0 , m]

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JACOBI METHOD

p = 3, q = 4

$$\begin{vmatrix} 1 & & & & & \\ & 1 & & & & \\ & & c & -s & & \\ & & s & c & & \\ & & & & 1 & \\ & & & & & 1 \end{vmatrix} \begin{vmatrix} x & x & x & x & x & x \\ x & x & x & x & x & x \\ x & x & x & * & x & x \\ x & x & * & x & x & x \\ x & x & x & x & x & x \\ x & x & x & x & x & x \end{vmatrix} \begin{vmatrix} 1 & & & & & \\ & 1 & & & & \\ & & c & s & & \\ & & -s & c & & \\ & & & & 1 & \\ & & & & & 1 \end{vmatrix} = \begin{vmatrix} x & x & x & x & x & x \\ x & x & x & x & x & x \\ x & x & x & 0 & x & x \\ x & x & 0 & x & x & x \\ x & x & x & x & x & x \\ x & x & x & x & x & x \end{vmatrix} \begin{matrix} \\ \\ \leftarrow \\ \leftarrow \\ \\ \\ \end{matrix}$$

J^T A J ↑ ↑

left transform/

rowp = c*A(p,:) -s*A(q,:) ← cost = O(n) 4n
rowq = s*A(p,:) +c*A(q,:) 4n

right 4n + 4n

16n ops altogether

times n(n-1)/2 == ~ $8 n^3$ per iteration

convergence is quadratic

IMPORTANT: ONLY ROWS p and q AND COLUMNS p and q CHANGE IN THIS TRANSFORMATION

$$\begin{vmatrix} c & -s \\ s & c \end{vmatrix} \begin{vmatrix} d & e \\ e & f \end{vmatrix} \begin{vmatrix} c & s \\ -s & c \end{vmatrix} = \ ?$$

More general case for J^T

$$\begin{vmatrix} 1 & & & & \\ & 1 & & & \\ & & c & 0 & -s \\ & & 0 & 1 & 0 \\ & & s & 0 & c \\ & & & & & 1 \end{vmatrix}$$

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$A_0 = A - Diag(A)$

2x2 matrix

Frob norm of 2x2 matrix A([p,q],[p,q]) is preserved!