✍ 2 *Prove that Gram-Schmidt can be completed iff the x_i's are linearly independent.*

Solution:

We will show that Gram-Schmidt breaks down iff the x_i's are linearly dependent.

The only way in which GS can break down is if $r_{jj} = \|\hat{q}\|_2$ in line 7 is zero.

The main observation is that the vector $\hat{q}$ at the end of the loop starting in line 4 (ending in line 6) is a linear combination of the x_i's for $i = 1, \cdots, j-1$. [simple proof by induction – omitted.]

☐

✍ 3 *Cost of Gram-Schmidt?*

Solution: *Step j of the algorithm costs : $(j-1) \times 2m$ operations for line 3, + $(j-1) \times 2m$ operations for loop in line 4 + $3m$ operations in Lines 7 and 8 together. Total for step j =*

$c_j = (4j - 1)m$. *Total over the* n *columns* $= T(n) = (2n^2 + n)m \approx 2n^2m$.

Note: this is linear in m *(number of rows) and quadratic in* n *(number of columns).* $\square$

✍5 *What is the cost of solving a linear system with the QR factorization?*

Solution: *According to the previous question we have a cost of* $2n^3$ *for the factorization (since* $m = n$*), to which we need to add the cost of solving a triangular solve* $O(n^2)$ *and the cost for computing* Q^Tb *which is again* $O(n^2)$. *In the end the cost is dominated by the QR factorization which is* $2n^3$. *This is 3 times more expensive than GE.* $\square$