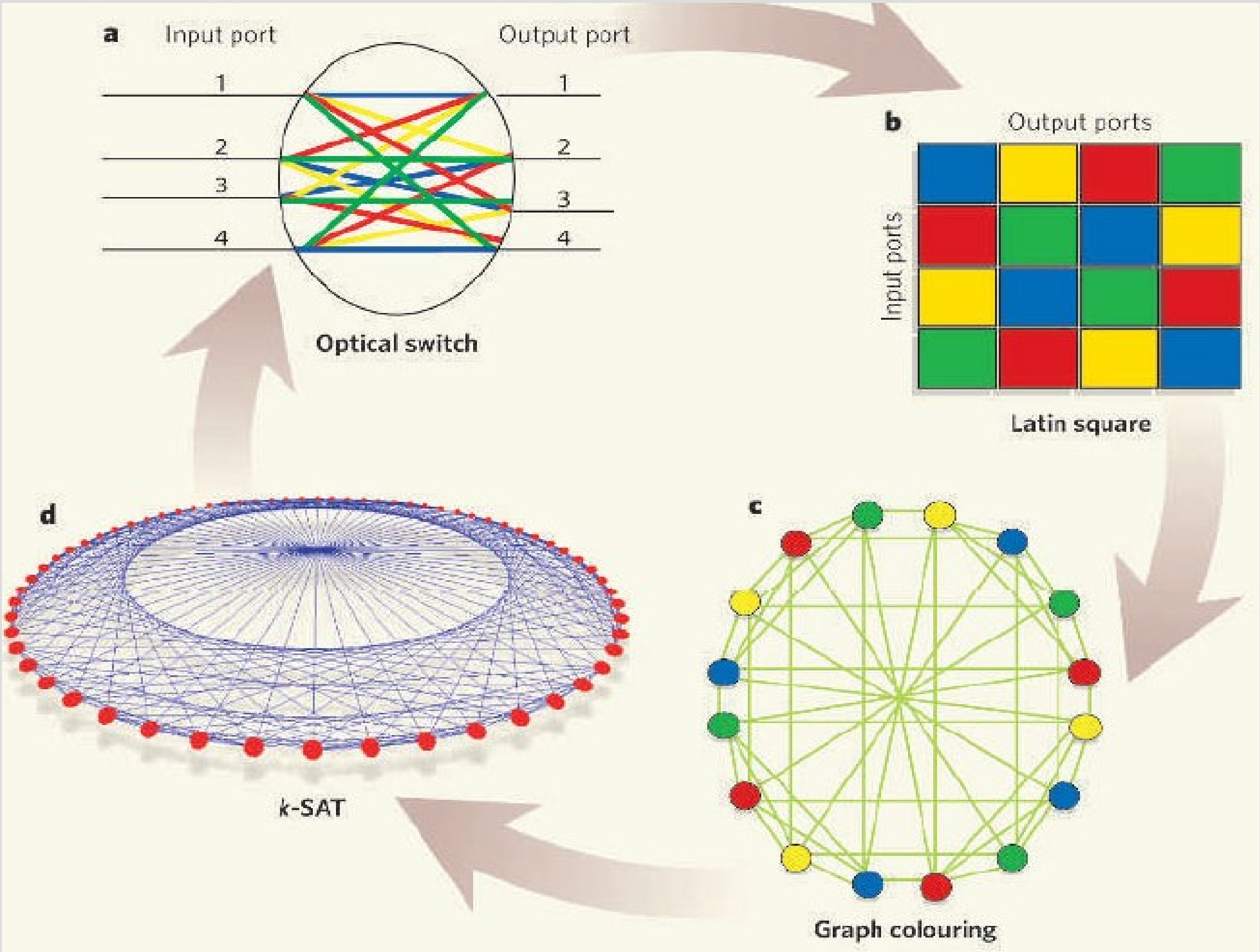
Constraint sat. prob. (Ch. 6)



Announcements

Writing 3 assigned Wed.

- -Find papers (like writ 2) for project
- -scholar.google.com is your friend!

CSP

A <u>constraint satisfaction problem</u> is when there are a number of variables in a domain with some restrictions

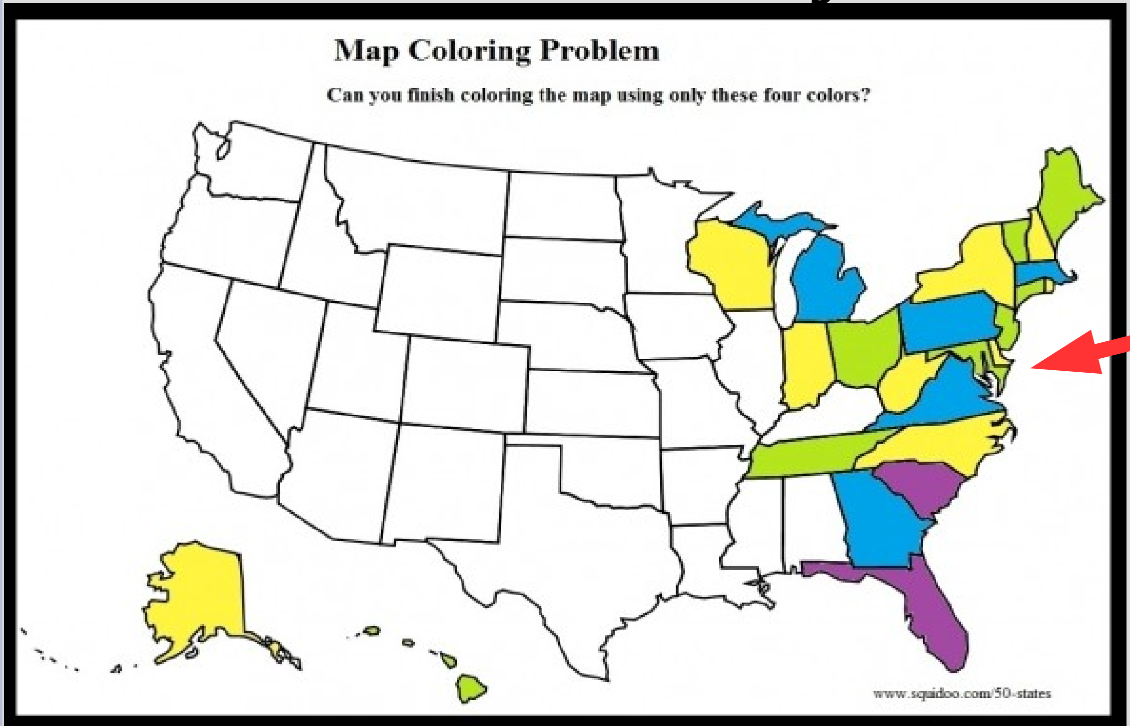
A <u>consistent</u> assignment of variables has no violated constraints

A <u>complete</u> assignment of variables has no unassigned variables

(A solution is complete and consistent)

CSP

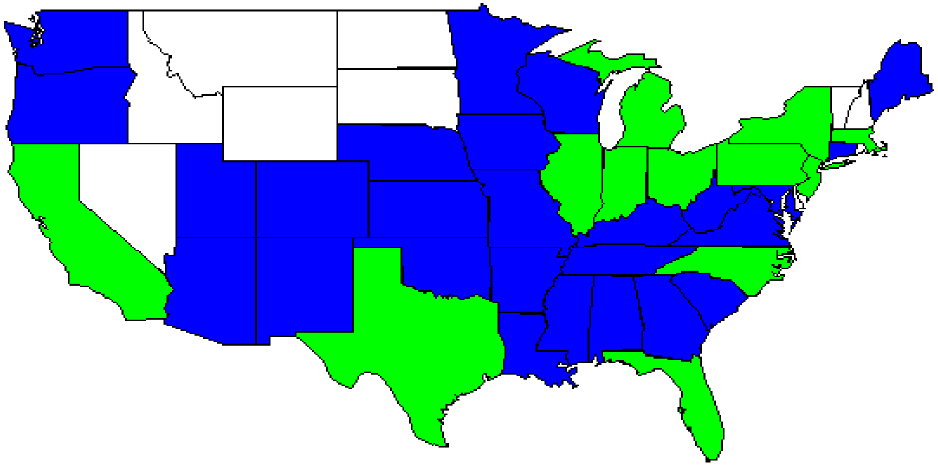
Map coloring is a famous CSP problem
Variables: each state/country
Domain: {yellow, blue, green, purple} (here)
Constraints: No adjacent variables same color

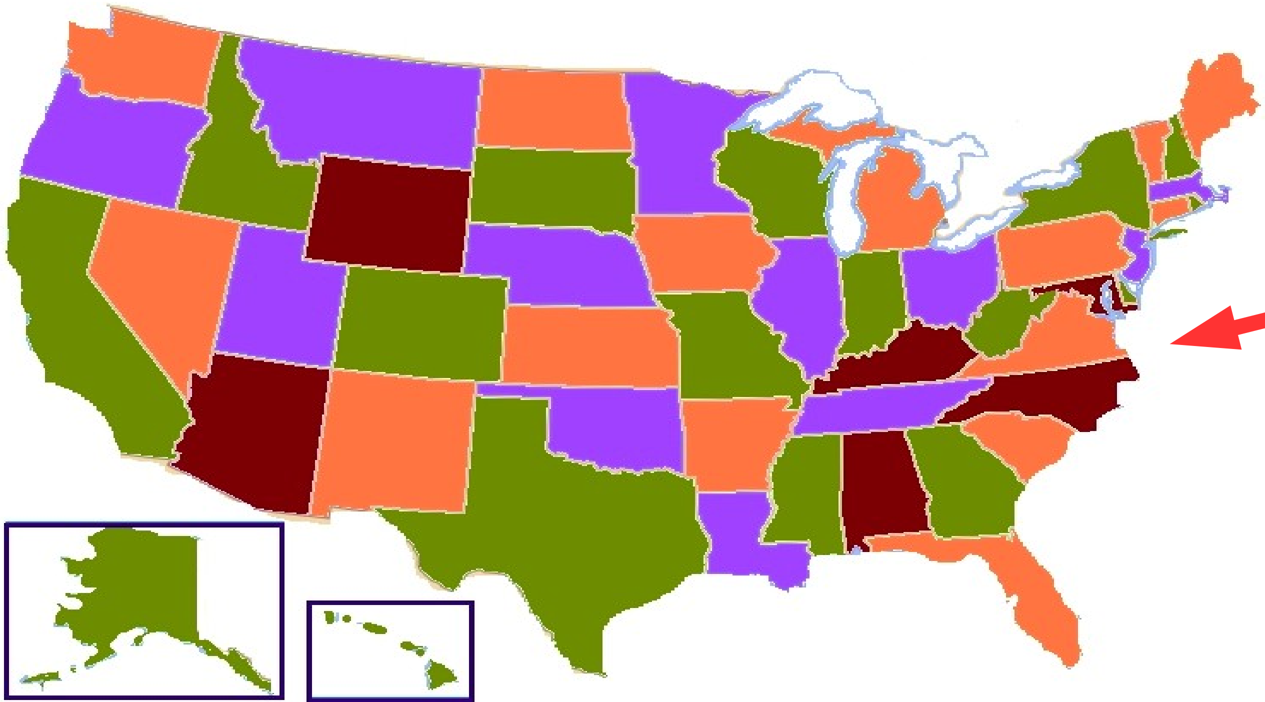


Consistent
but partial

CSP

partial and
not consistent

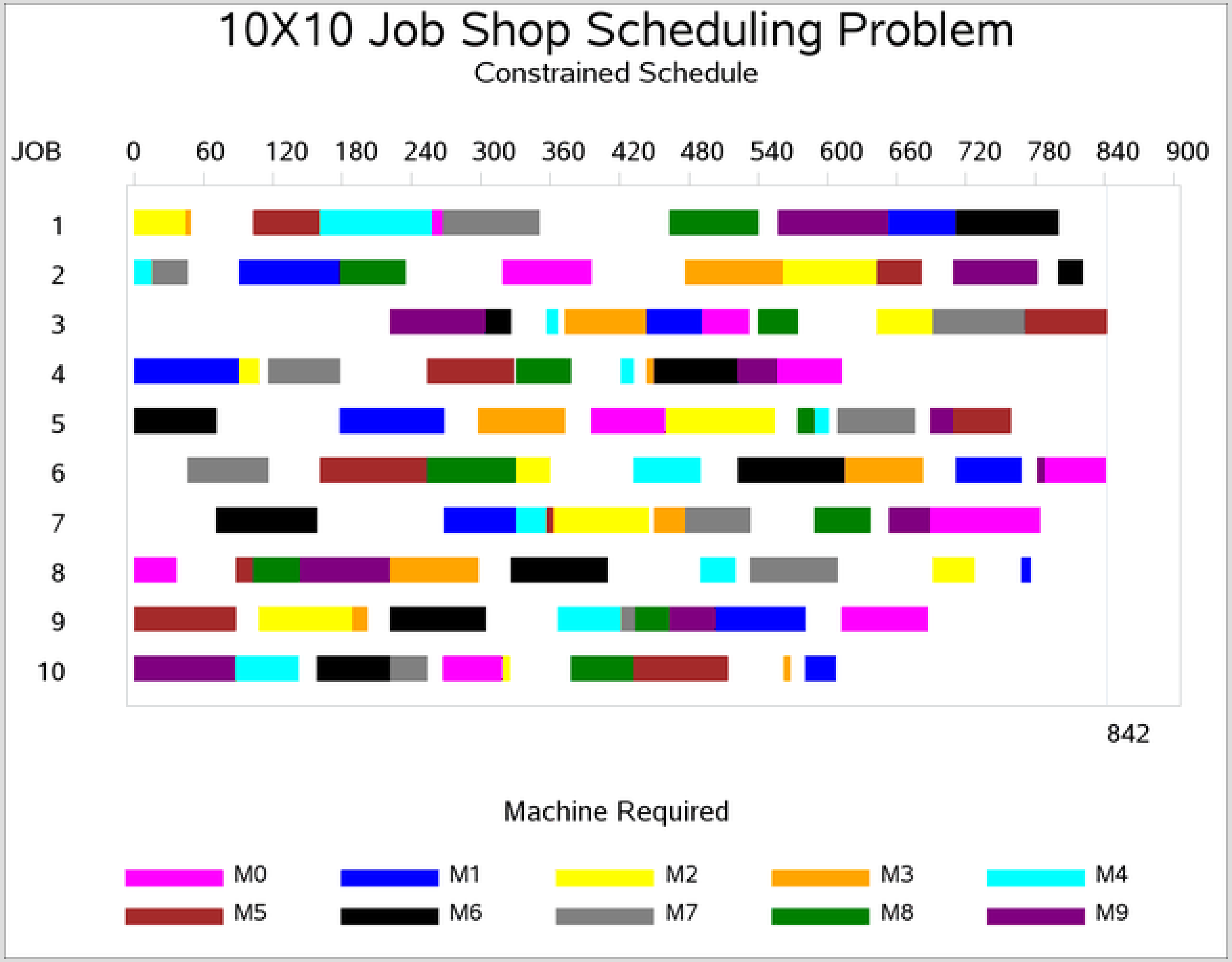




Consistent and
complete

CSP

Another common use of CSP is job scheduling



CSP

Suppose we have 3 jobs: J_1, J_2, J_3

If J_1 takes 20 time units to complete, J_2 takes 30 and J_3 takes 15 <u>but</u> J_1 must be done before J_3

How to write this as a boolean expression?
(jobs cannot be scheduled at the same time)

CSP

Suppose we have 3 jobs: J_1, J_2, J_3

If J_1 takes 20 time units to complete, J_2 takes 30 and J_3 takes 15 <u>but</u> J_1 must be done before J_3

We can represent this as (**<u>and</u>** them together):

J_1 & J_2: ($J_1 + 20 \leq J_2$ **<u>or</u>** $J_2 + 30 \leq J_1$)

J_1 & J_3: ($J_1+20 \leq J_3$)

J_2 & J_3: ($J_2 + 30 \leq J_3$ **<u>or</u>** $J_3 + 15 \leq J_2$)

Types of constraints

A unary constraint is for a single variable (i.e. J_1 cannot start before time 5)

Binary constraints are between two variables (i.e. J_1 starts before J_2)

All constraints can be broken down into using only binary and unary

Types of constraints

K-consistency is:
For any consistent sets size (k-1), there exists a valid value for any other variable (not in set)

1-consistency: All values in the domain satisfy the variable's unary constraints
2-consistency: All binary values are in domain
3-consistency: Given consistent 2 variables, there is a value for a third variable(i.e. if {A,B} is consistent, then exists C s.t. {A,C}&{B,C})

Types of constraints

For example, 1-consistent means you can pick 0 consistent variables (if you pick nothing it is always consistent) then any assignment to a new variable is also consistent

This boils down to saying you can pick any valid pick of a single variable in isolation

In other words, you satisfy the unary constraints

Types of constraints

2-consistent means you pick a valid value from the domain for one variable and see if there is <u>any</u> valid assignment for a second var

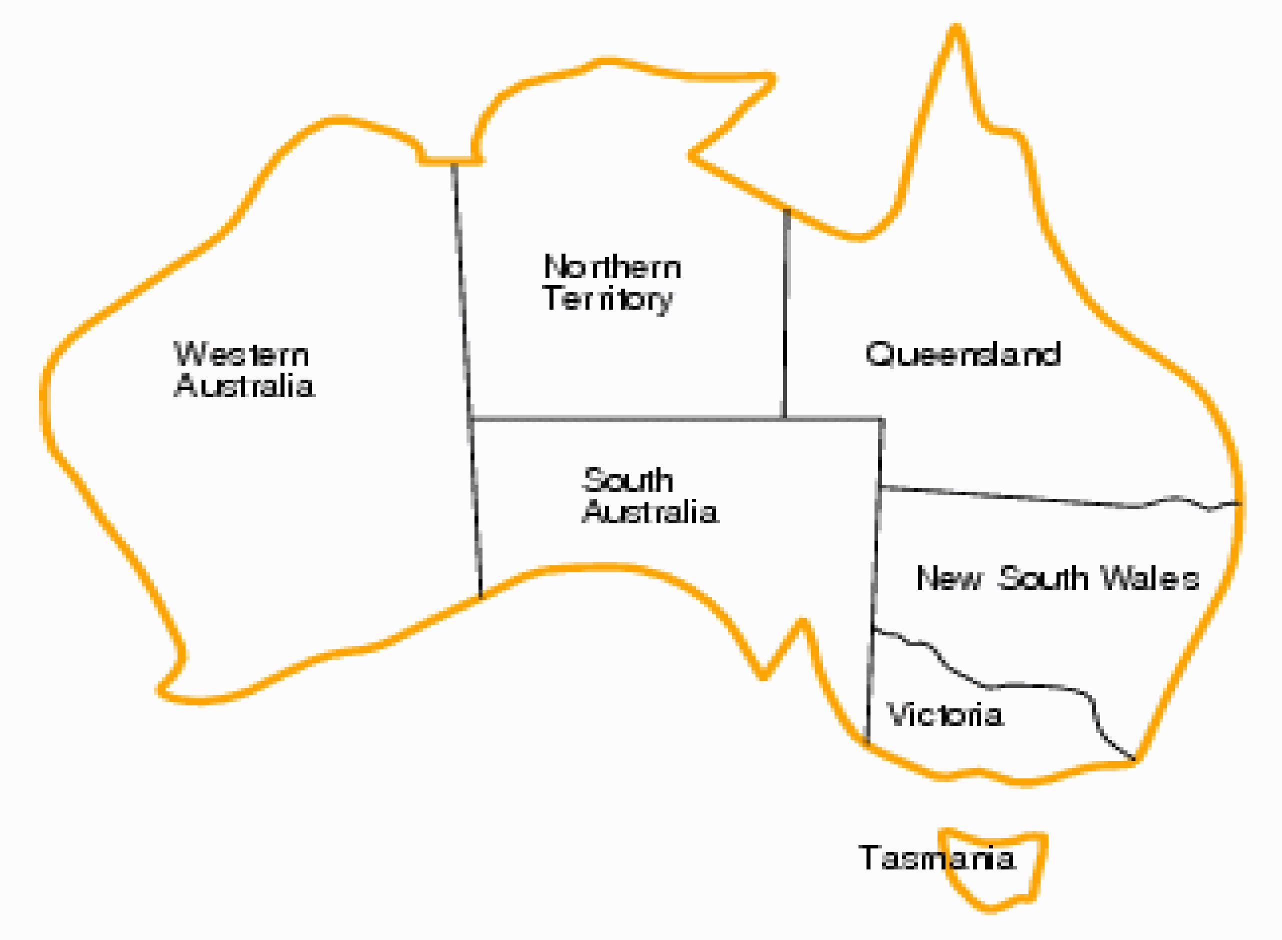
3-consistent means you pick a valid pair of values for 2 variables and see if there is <u>any</u> valid assignment for a third variable

If you are unable to find a valid assignment for the last variable, it is not consistent

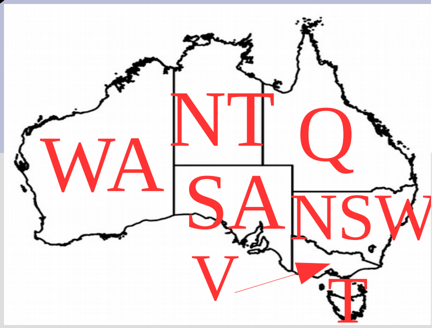
Types of constraints

Rules: 1. Tasmania cannot be red
2.Neighboring providences cannot share colors

2 Colors:
red
green



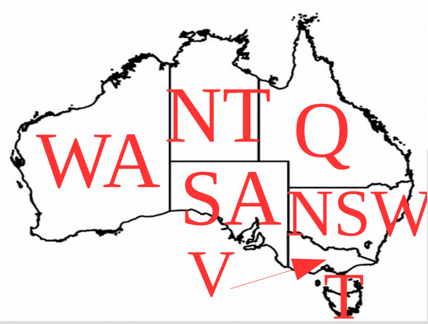
Types of constraints



WA = {red, green}
NT = {red, green}
Q = {red, green}
SA = {red, green}
NSW = {red, green}
V = {red, green}
T = {red, green}

Not 1-consistent as we need T to not be red
(i.e. rule #2 eliminates T=red)

Types of constraints

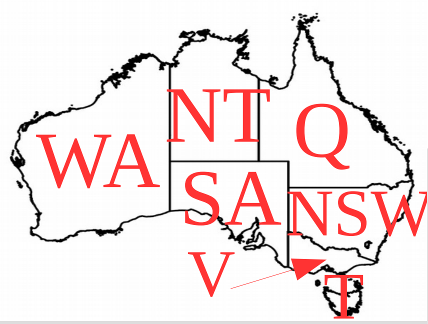


WA = NT = Q = SA = NSW = V
= {red, green}
T = {green}

1-consistent now

Also 2-consistent, for example:
Pick WA as “set k-1”, then try to pick NT...
If WA=green, then we can make NT=red
if WA=red, NT=green (true for all pairs)

Types of constraints



WA = NT = Q = SA = NSW = V
= {red, green}
T = {green}

Not 3-consistent!

Pick (WA, SA) and add NT... If NT=green, will not work with either: (WA=red,SA=green) or (WA=green,SA=red)... NT=red also will not work, so NT's domain is empty and not 3-cons.

Types of constraints

Try to do this job problem with: J1, J2 and J3
(Domains are positive integers)
Jobs cannot overlap
J3 takes 3 time units
J2 takes 2 time units
J1 takes 1 time unit
J1 must happen before J3
J2 cannot happen at time 1
All jobs must finish by time 7
(i.e. you can start J2 at time 5 but not at time 6)

Applying constraints

We can repeatedly apply our constraint rules to shrink the domain of variables (we just shrunk NT's domain to nothing)

This reduces the size of the domain, making it easier to check:
- If the domain size is zero, there are no solutions for this problem
- If the domain size is one, this variable must take on that value (the only one in domain)

Applying constraints

AC-3 checks all 2-consistency constraints:

1. Add all binary constraints to queue
2. Pick a binary constraint (X_i, Y_j) from queue
3. If x in domain(X_i) and no consistent y in domain(Y_j), then remove x from domain(X_i)
4. If you removed in step 3, update all other binary constraints involving X_i (i.e. (X_i, X_k))
5. Goto step 2 until queue empty

Applying constraints

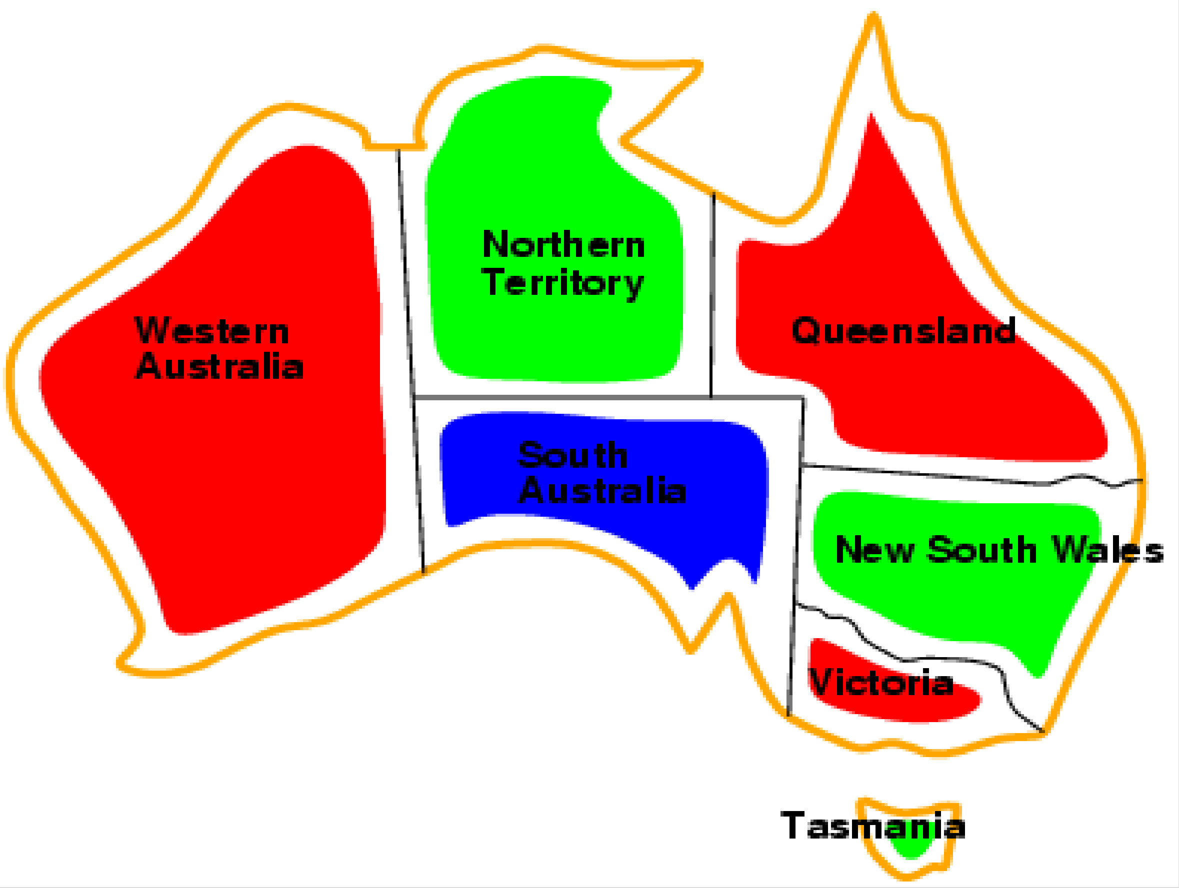
Some problems can be solved by applying constraint restrictions (such as sudoku) (i.e. the size of domain is one after reduction)

Harder problems this is insufficient and we will need to search to find a solution

Which is what we will do... now

CSP vs. search

Let us go back to Australia coloring:



How can you color using search techniques?

CSP vs. search

We can use an incremental approach:

State = currently colored provinces (and their color choices)

Action = add a new color to any province that does not conflict with the constraints

Goal: To find a state where all provinces are colored

CSP vs. search

Is there a problem?

CSP vs. search

Is there a problem?

Let d = domain size (number of colorings),
n = number of variables (provinces)

The number of leaves are $n! * d^n$

However, there are only d^n possible states
in the CSP so there must be a lot of duplicate
leaves (not including mid-tree parts)

CSP vs. search

CSP assumes one thing general search does not: the order of actions does not matter

In CSP, we can assign a value to a variable at any time and in any order without changing the problem (all we care about is the end state)

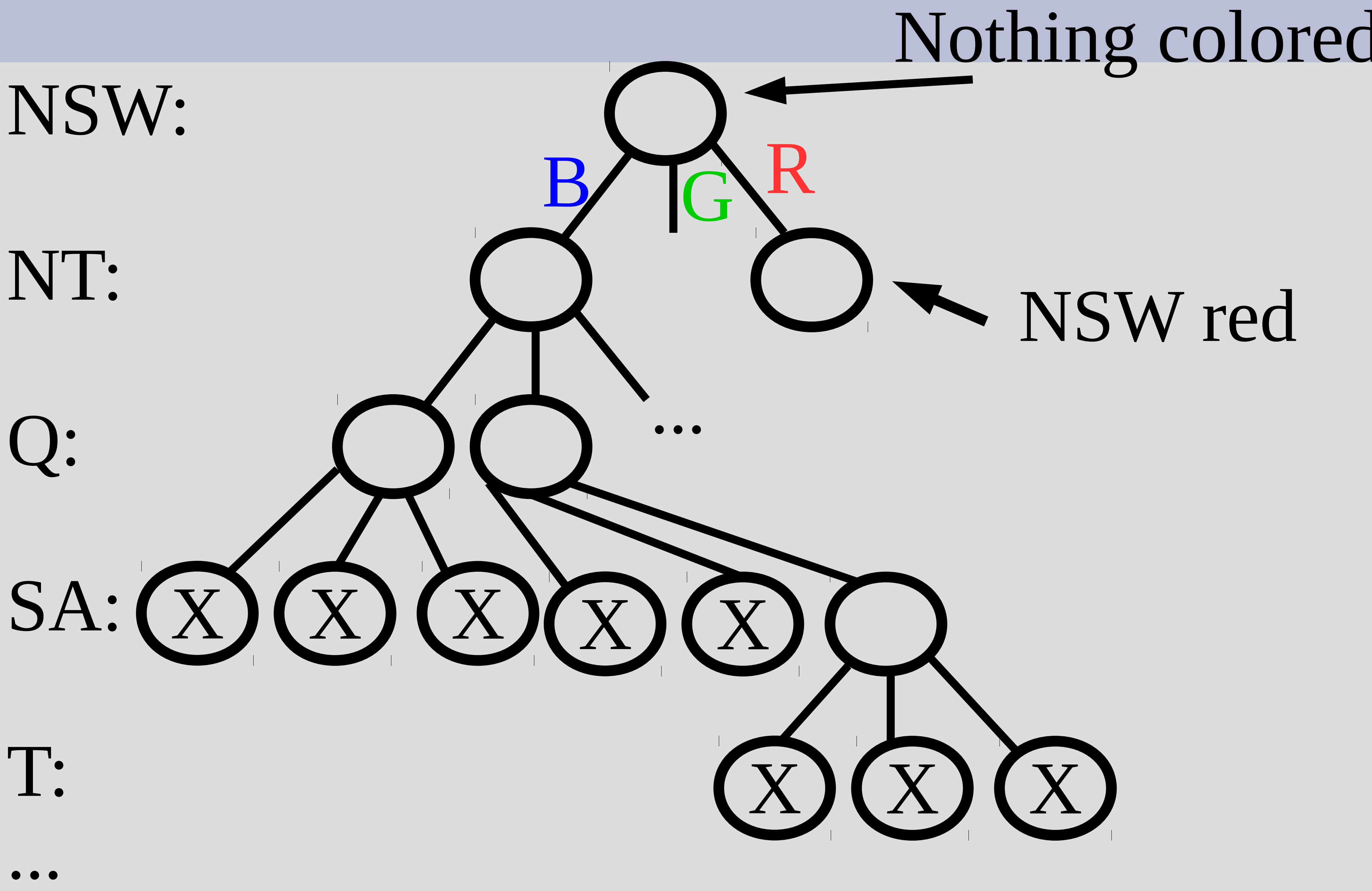
So all we need to do is limit our search to one variable per depth, and we will have a match with CSP of d^n leaves (all combinations)

CSP vs. search

Let's apply CSP modified DFS on Australia:
(assign values&variables in alphabetical order)

1^{st}: blue
2^{nd}: green
3^{rd}: red

CSP vs. search



CSP vs. search

STOP PICKING BLUE EVERY TIME!!!!