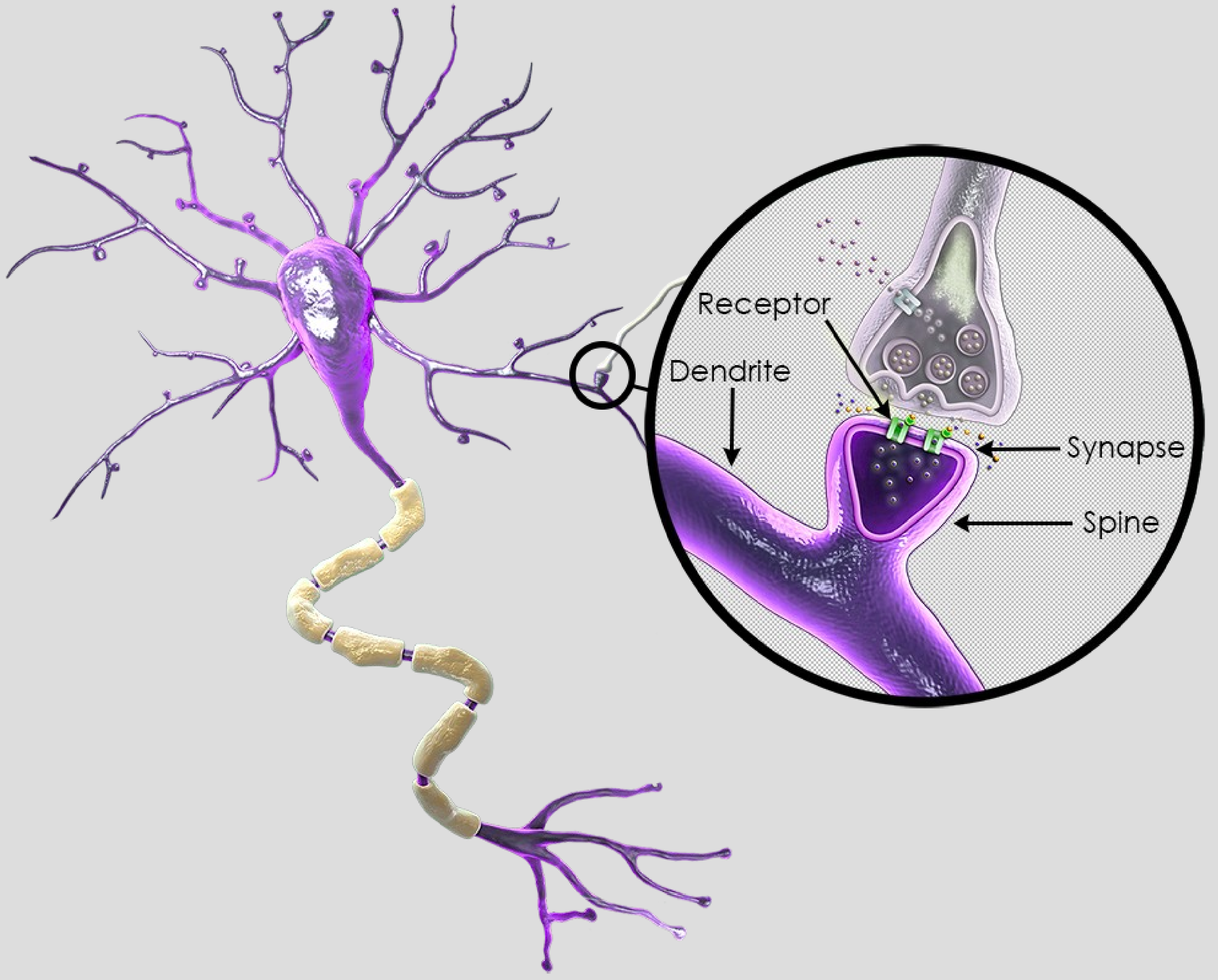
# Neural networks (Ch. 12)



# Biology: brains

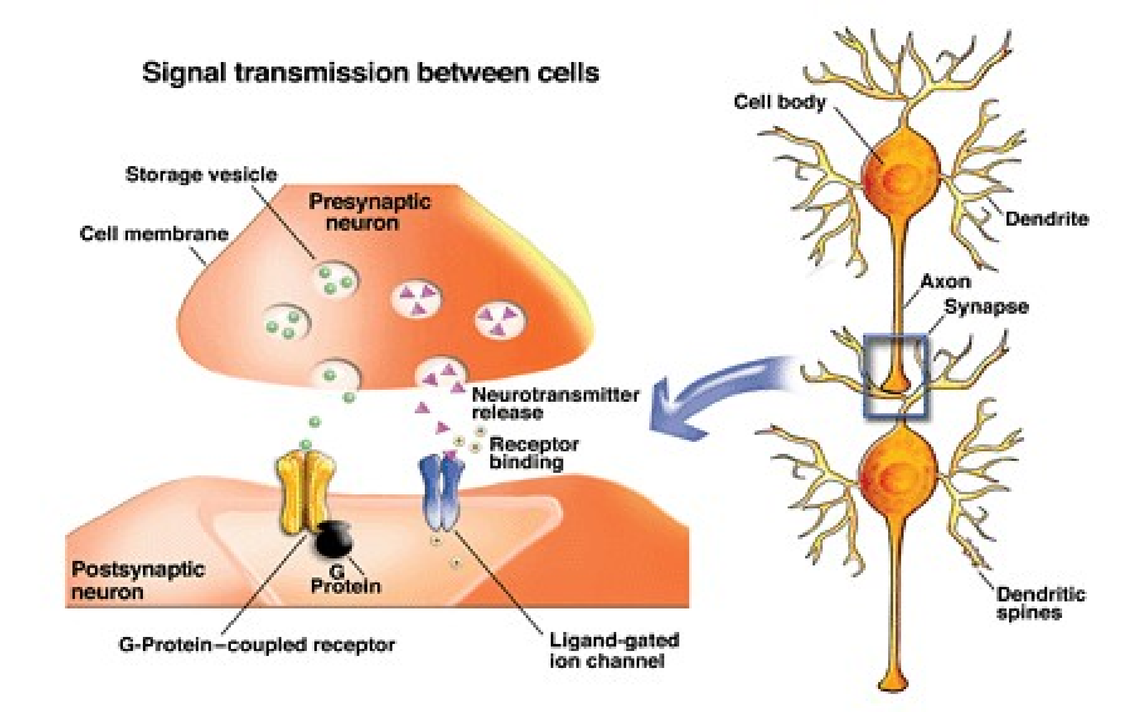
Computer science is fundamentally a creative process: building new & interesting algorithms

As with other creative processes, this involves mixing ideas together from various places

Neural networks get their inspiration from how brains work at a fundamental level (simplification... of course)

# Biology: brains

(Disclaimer: I am **not** a neuroscience-person)
Brains receive small chemical signals at the "input" side, if there are enough inputs to "activate" it signals an "output"



# Biology: brains

An analogy is sleeping: when you are asleep, minor sounds will not wake you up

However, specific sounds in combination with their volume will wake you up

# Biology: brains

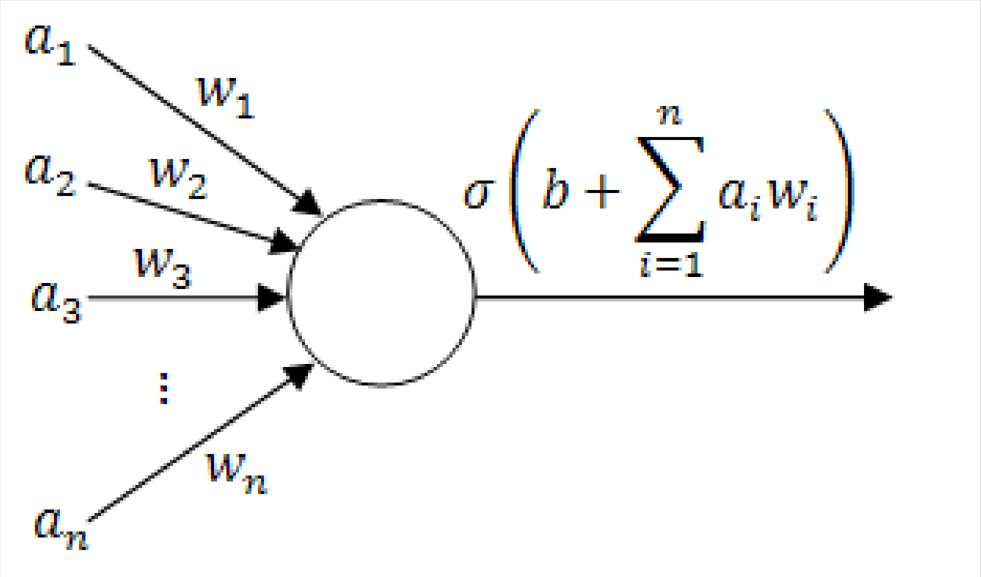
Other sounds might help you go to sleep (my majestic voice?)

Many babies tend to sleep better with “white noise” and some people like the TV/radio on

# Neural network: basics

Neural networks are connected nodes, which can be arranged into layers (more on this later)

First is an example of a perceptron, the most simple NN; a single node on a single layer



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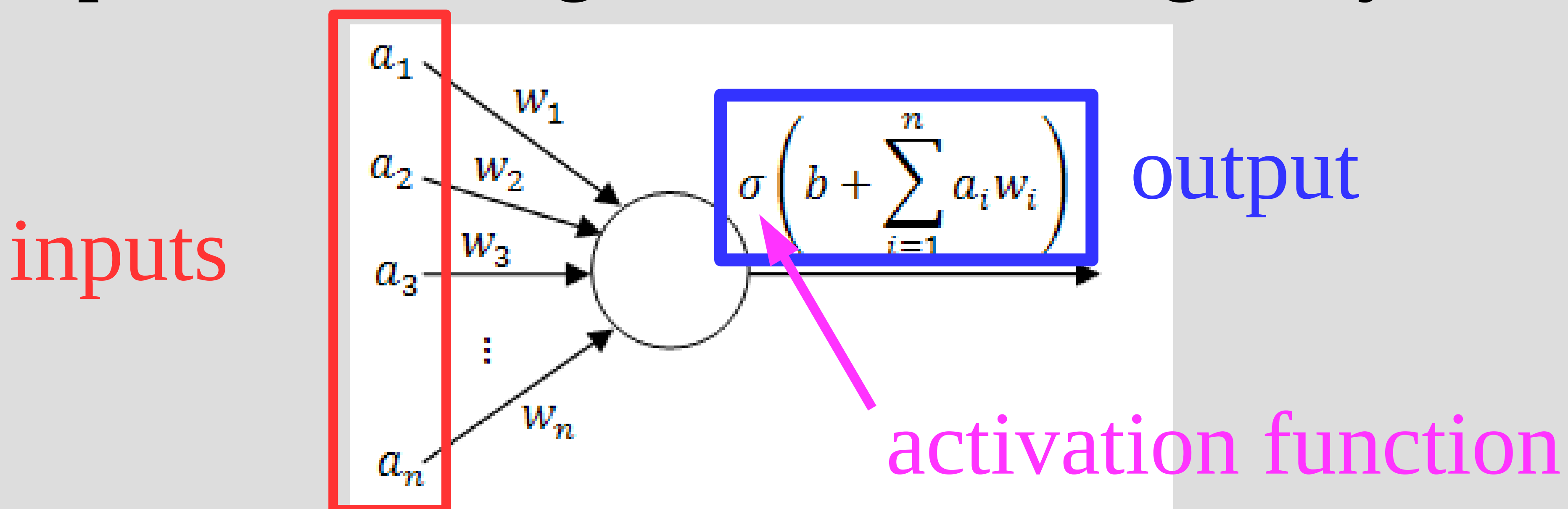


# Mammals

Let's do an example with mammals...

First the definition of a mammal (wikipedia):

Mammals [posses]:
(1) a neocortex (a region of the brain),
(2) hair,
(3) three middle ear bones,
(4) and mammary glands

# Mammals

Common mammal misconceptions:
(1) Warm-blooded
(2) Does not lay eggs

Let's talk dolphins for one second.
http://mentalfloss.com/article/19116/if-dolphins-are-mammals-and-all-mammals-have-hair-why-arent-dolphins-hairy

Dolphins have hair (technically) for the first week after birth, then lose it for the rest of life ... I will count this as “not covered in hair”

# Perceptrons

Consider this example: we want to classify whether or not an animal is mammal via a perceptron (weighted evaluation)

We will evaluate on:

1. Warm blooded? (WB) Weight = 2
2. Lays eggs? (LE) Weight = -2
3. Covered hair? (CH) Weight = 3

$$\text{If}(2 \cdot WB + -2 \cdot LE + 3 \cdot CH > 1) \Rightarrow Mammal$$

# Perceptrons

Consider the following animals:

Humans {WB=y, LE=n, CH=y}, mam=y

$2(1) + -2(-1) + 3(1) = 7 > 1 \ldots$ Correct!

Bat {WB=sorta, LE=n, CH=y}, mam=y

$2(0.5) + -2(-1) + 3(1) = 6 > 1 \ldots$ Correct!

What about these?

Platypus {WB=y, LE=y, CH=y}, mam=y

Dolphin {WB=y, LE=n, CH=n}, mam=y

Fish {WB=n, LE=y, CH=n}, mam=n

Birds {WB=y, LE=y, CH=n}, mam=n

# Perceptrons

Sadly, these weights classify dolphins wrong

But wait... what is the general form of:

$$w_1 x + w_2 \cdot y + w_3 \cdot z > c$$

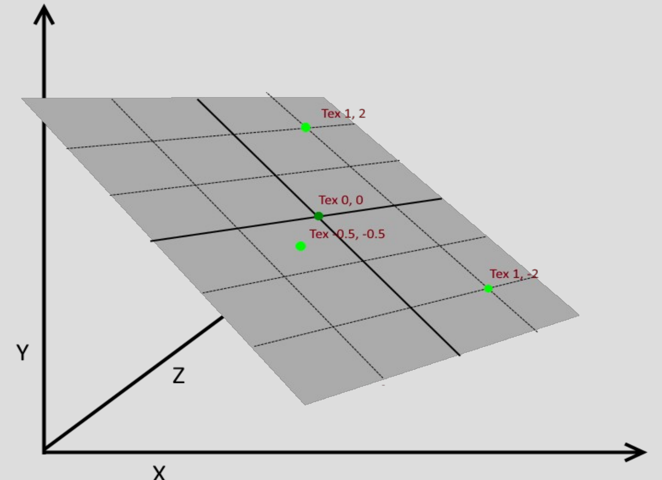
# Perceptrons

Sadly, these weights classify dolphins wrong

But wait... what is the general form of:

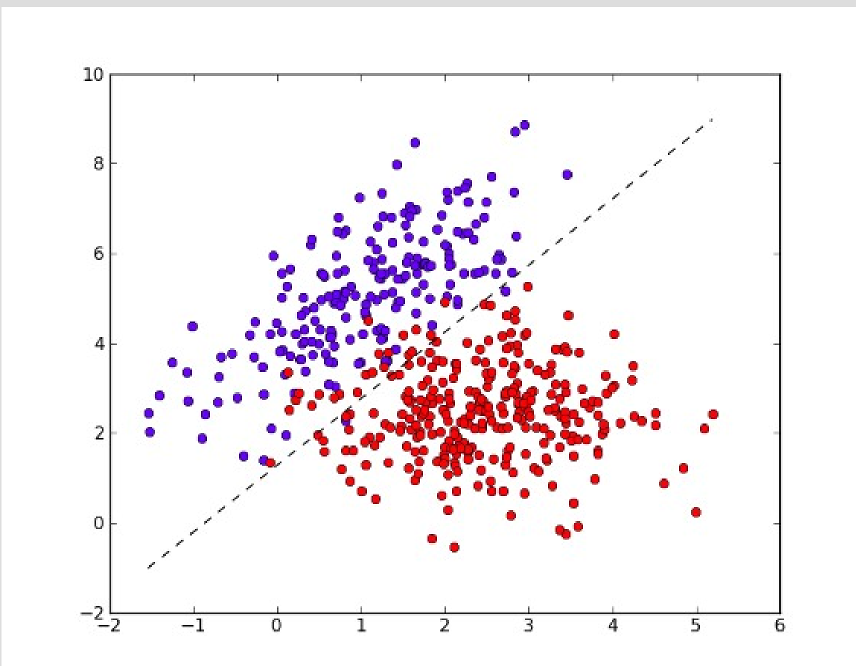
$$w_1 x + w_2 \cdot y + w_3 \cdot z > c$$

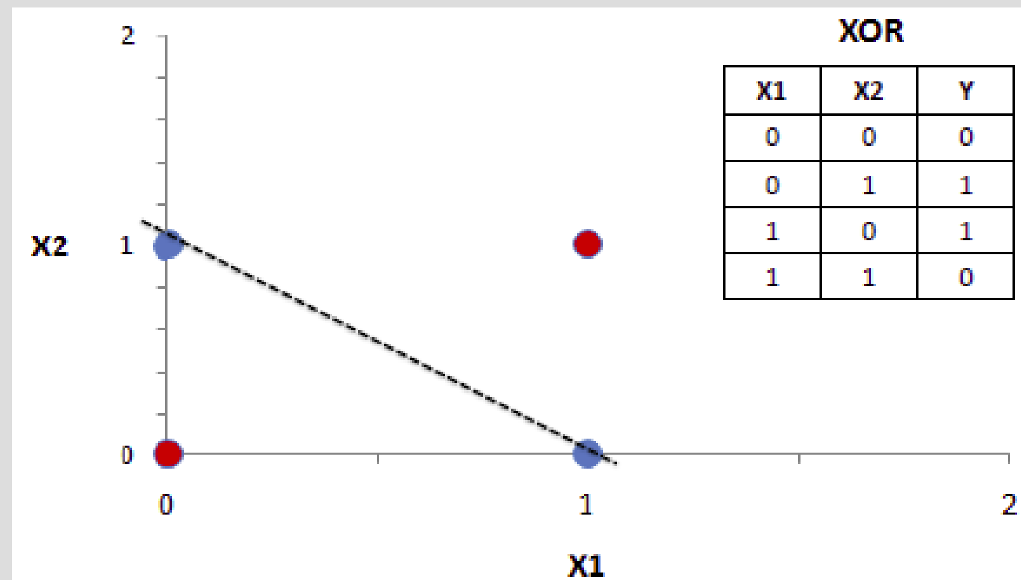
This is simply one side of a plane in 3D, so this is trying to classify all possible points using a single plane...



# Perceptrons

If we had only 2 inputs, it would be everything above a line in 2D, but consider XOR on right




There is no way a line can possibly classify this (limitation of perceptron)

# Neural network: feed-forward

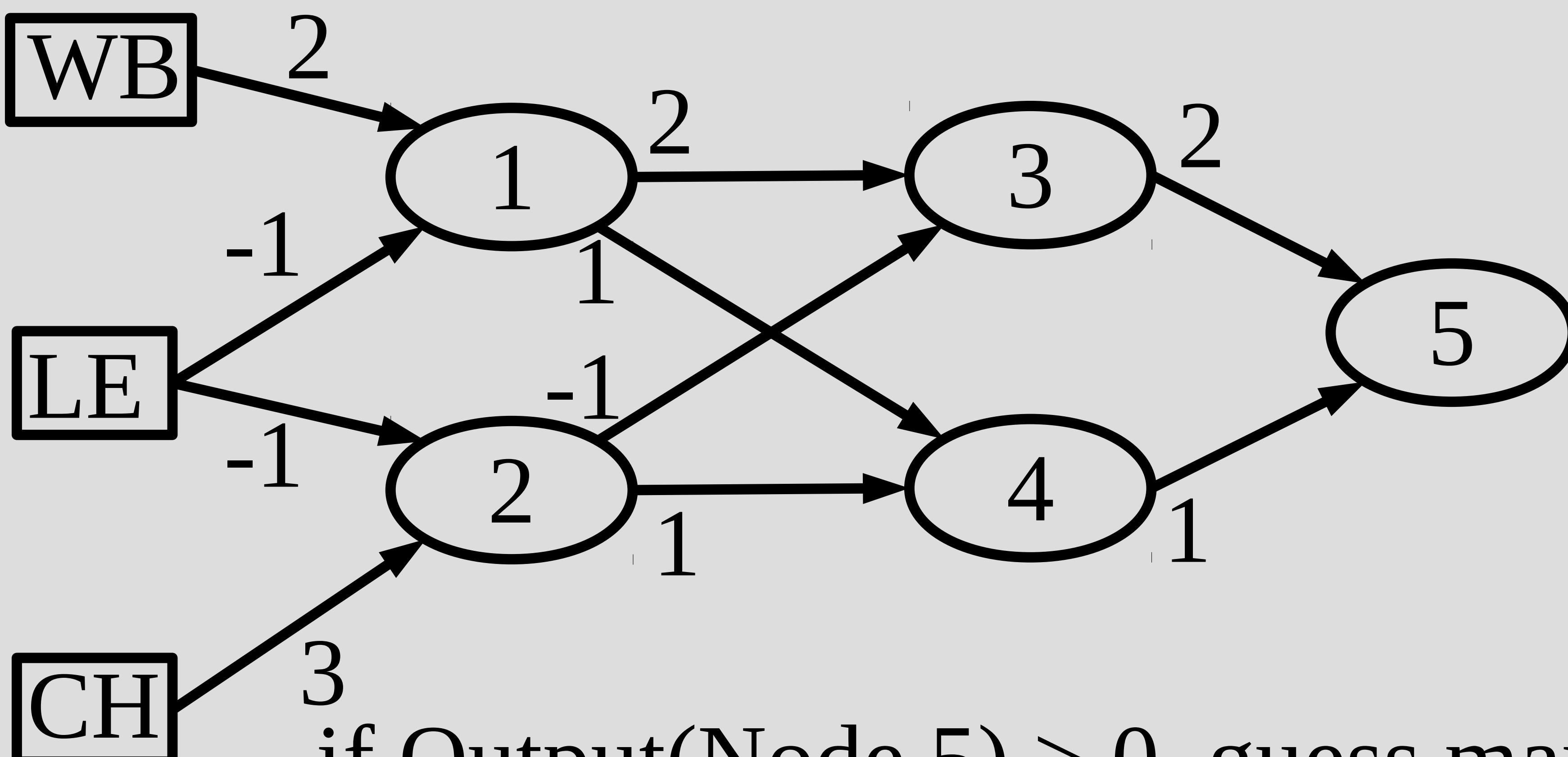
Today we will look at feed-forward NN, where information flows in a single direction

Recurrent networks can have outputs of one node loop back to inputs as previous

This can cause the NN to not converge on an answer (ask it the same question and it will respond differently) and also has to maintain some “initial state” (all around messy)

# Neural network: feed-forward

Let's expand our mammal classification to 5 nodes in 3 layers (weights on edges):

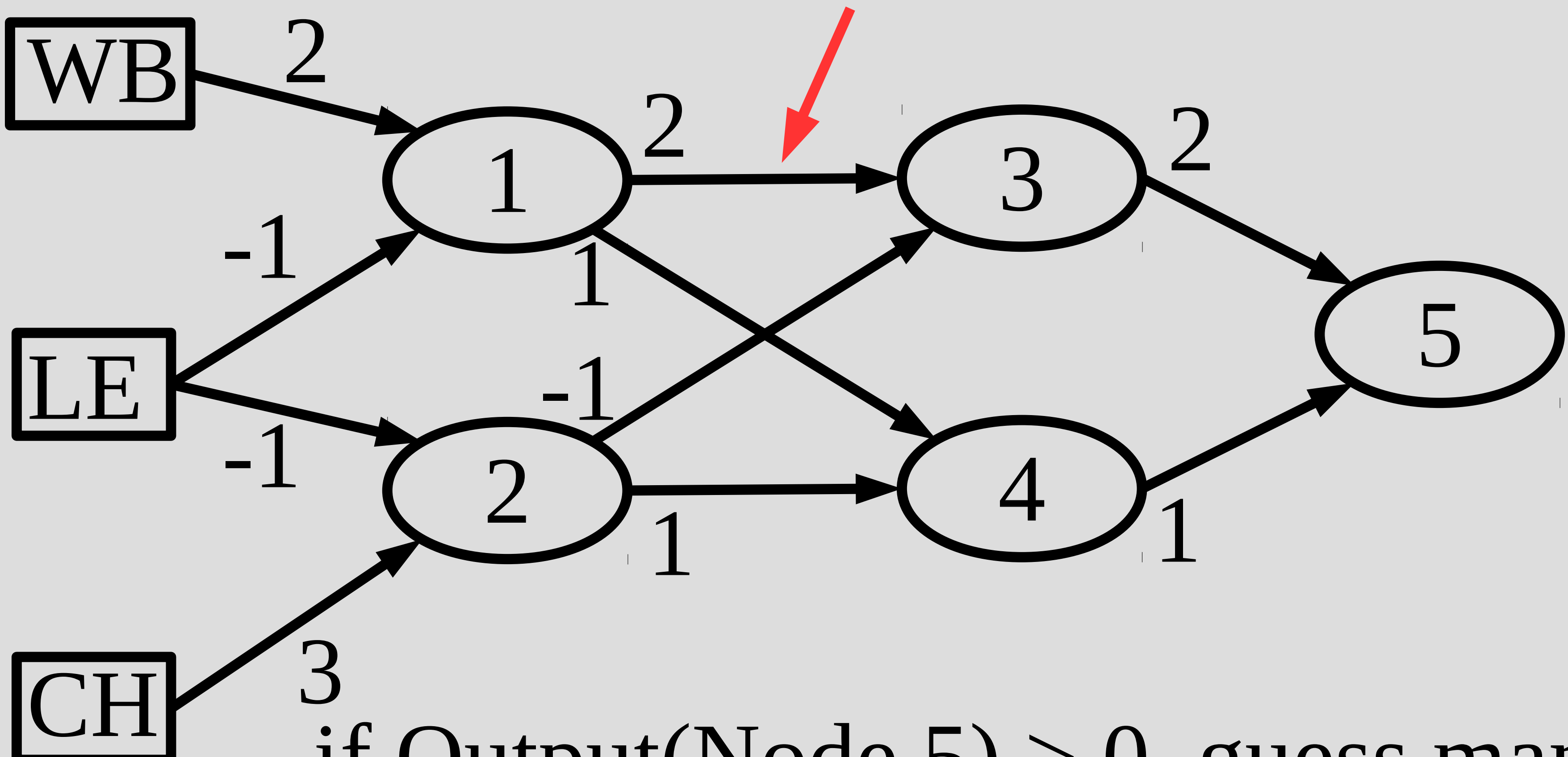



if Output(Node 5) > 0, guess mammal

# Neural network: feed-forward

You try Bat on this:{WB=0.5, LE=1, CH=1}

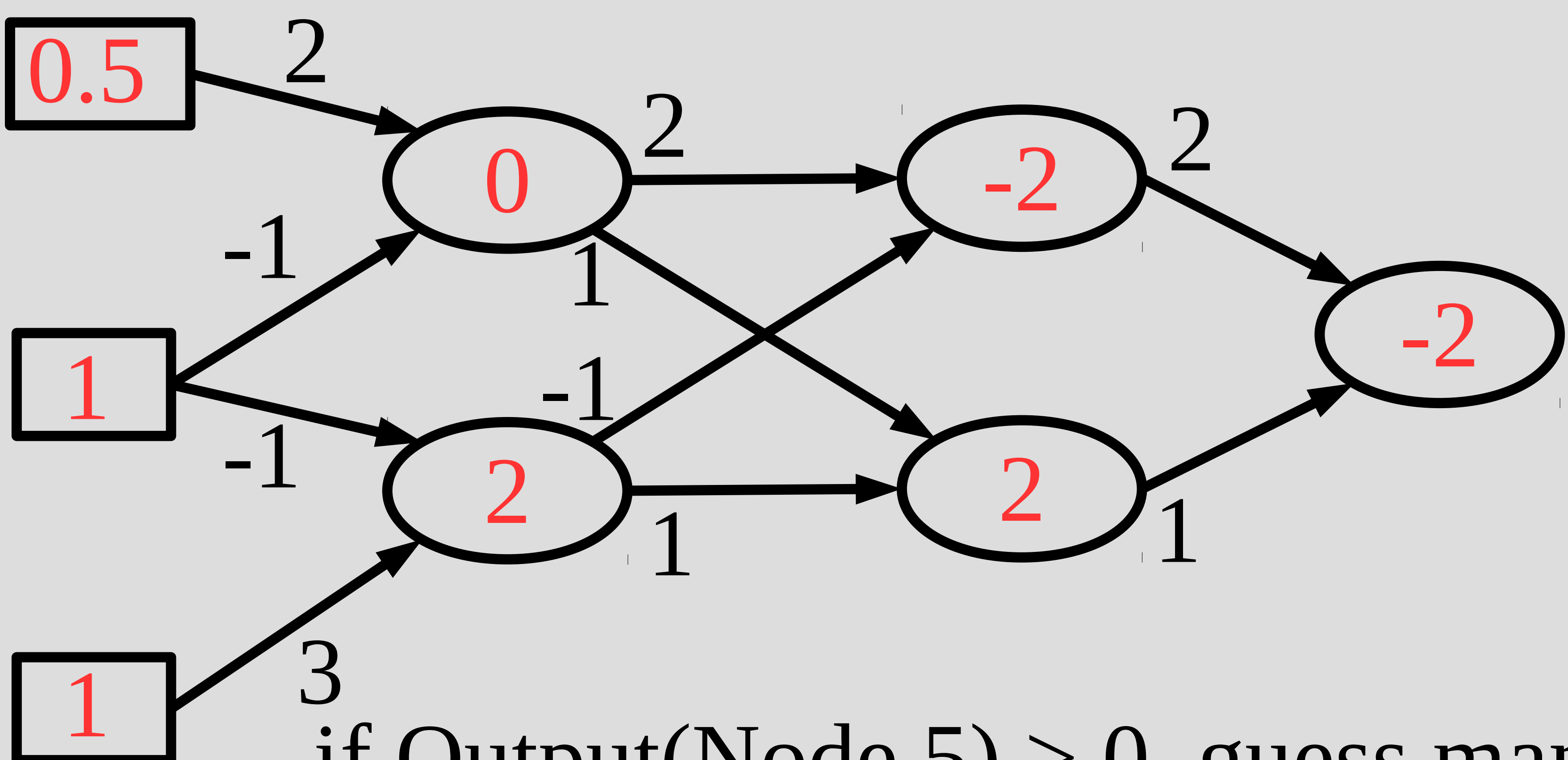
Assume (for now) output = node value




if Output(Node 5) > 0, guess mammal

# Neural network: feed-forward

Output is -2, so bats are not a mammal... Oops!




if Output(Node 5) > 0, guess mammal

# Neural network: feed-forward

In fact, this is no better than our 1 node NN

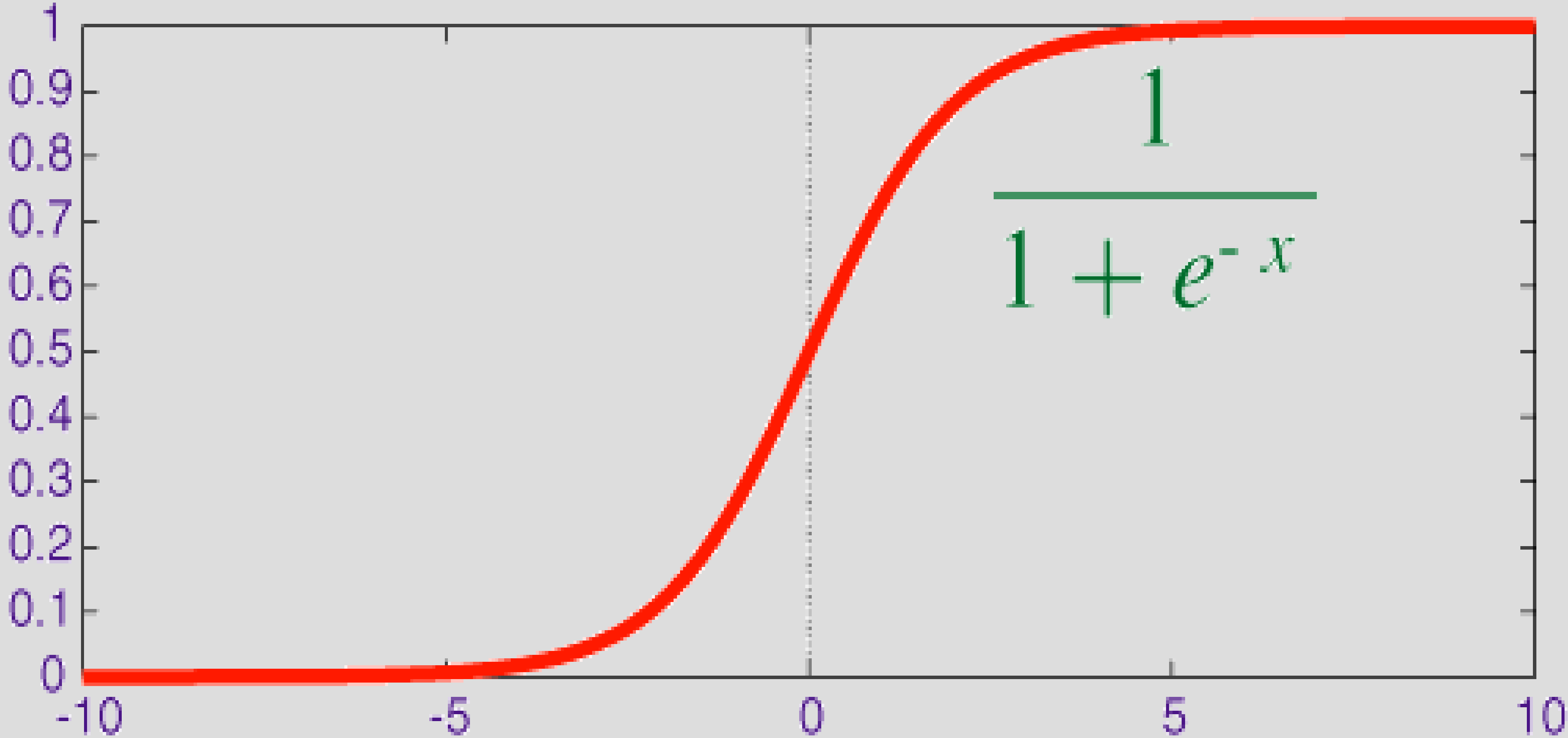
This is because we simply output a linear combination of weights into a linear function (i.e. if f(x) is linear... then f(f(x)) is also linear)

Ideally, we want a activation function that has a limited range so large signals do not always dominate

# Neural network: feed-forward

One commonly used function is the sigmoid:

$$S(x) = \frac{1}{1+e^{-x}}$$



# Back-propagation

The neural network is as good as it's structure and weights on edges

Structure we will ignore (more complex), but there is an automated way to learn weights

Whenever a NN incorrectly answer a problem, the weights play a “blame game”...

- Weights that have a big impact to the wrong answer are reduced

# Back-propagation

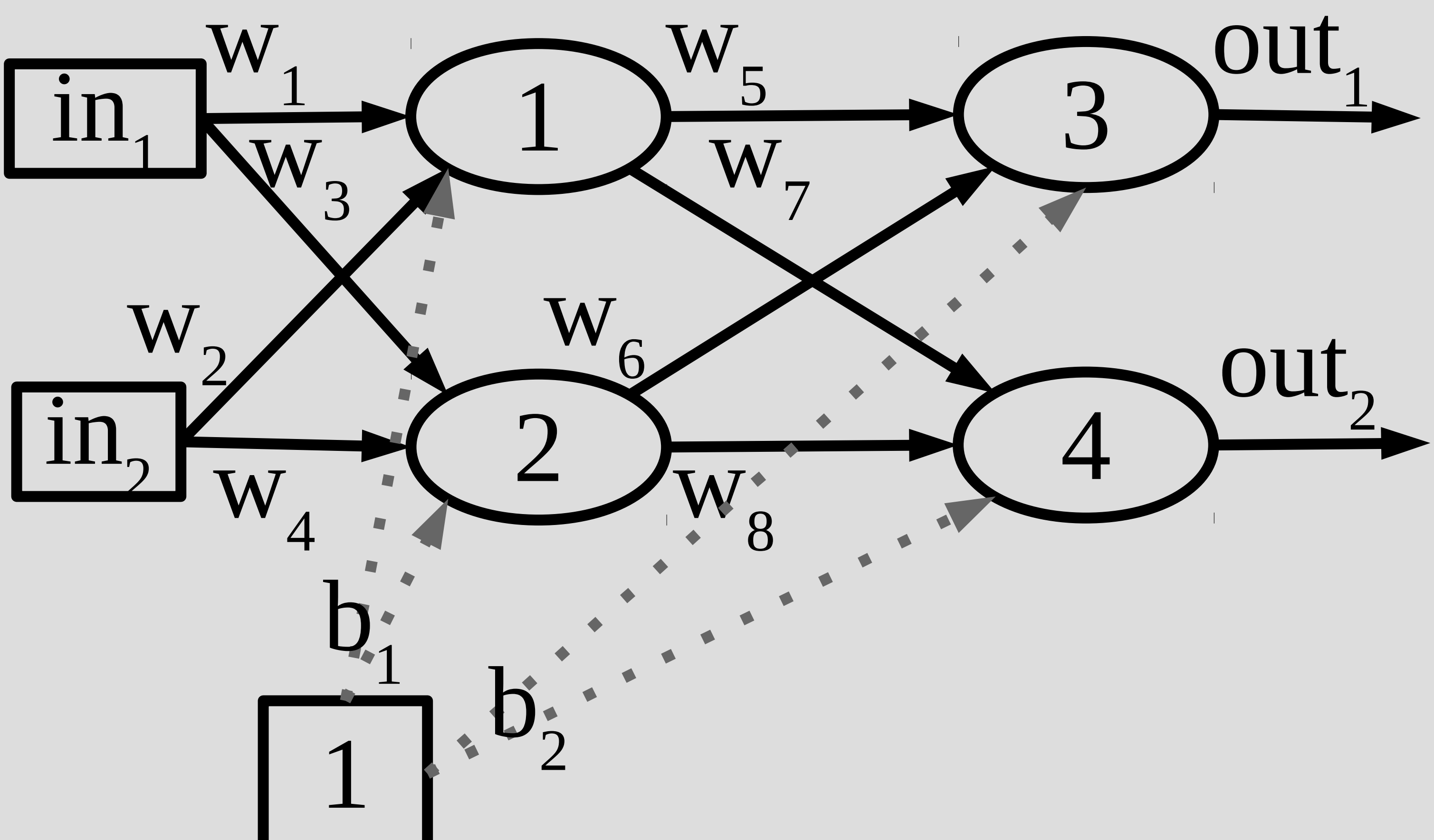
To do this blaming, we have to find how much each weight influenced the final answer

Steps:

1. Find total error
2. Find derivative of error w.r.t. weights
3. Penalize each weight by an amount proportional to this derivative

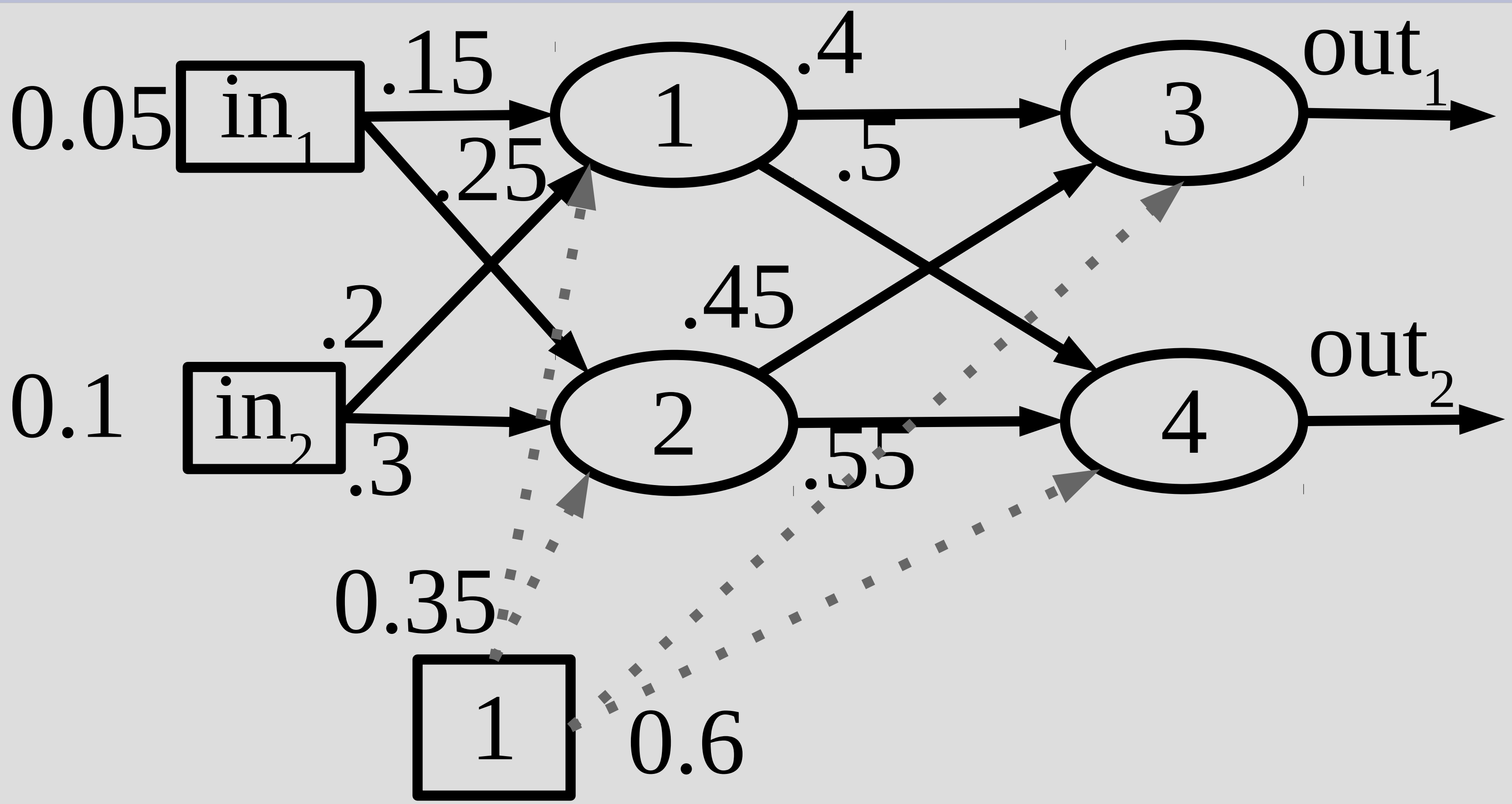
# Back-propagation

Consider this example: 4 nodes, 2 layers



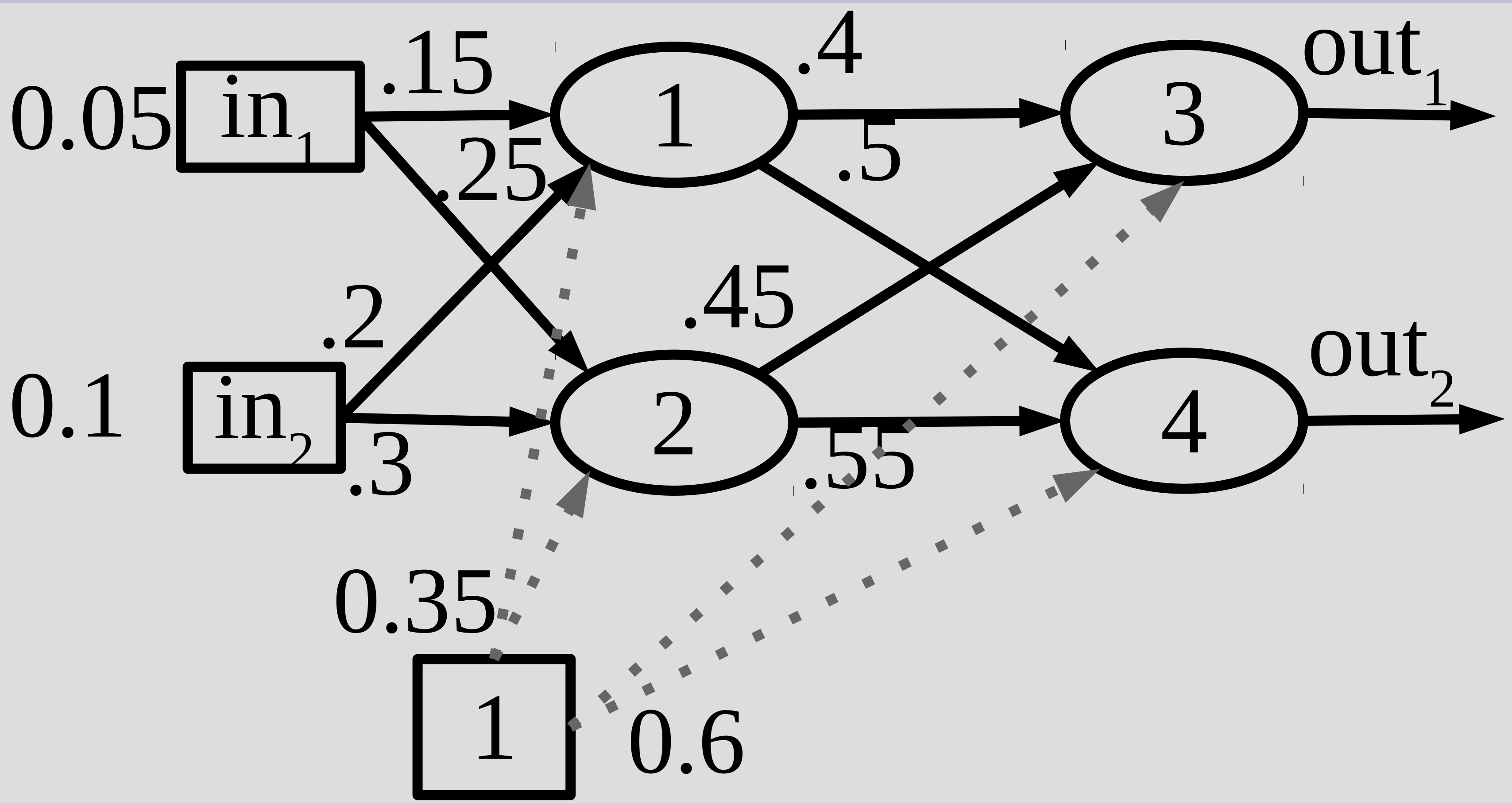

This node as a constant bias of 1

# Back-propagation




Node 1: 0.15*0.05 + 0.2*0.1 +0.35 as input thus it outputs (all edges) S(0.3775)=0.59327

# Back-propagation




Eventually we get: $out_1 = 0.7513$, $out_2 = 0.7729$

Suppose wanted: $out_1 = 0.01$, $out_2 = 0.99$

# Back-propagation

We will define the error as: $\frac{\sum_i (correct_i - output_i)^2}{2}$
(you will see why shortly)

Suppose we want to find how much $w_5$ is to blame for our incorrectness

We then need to find: $\frac{\partial Error}{\partial w_5}$
Apply the chain rule:

$$\frac{\partial Error}{\partial out_1} \cdot \frac{\partial S(In(N_3))}{\partial In(N_3)} \cdot \frac{\partial In(N_3)}{\partial w_5}$$

# Back-propagation

$$Error = \frac{\sum_i (correct_i - output_i)^2}{2}$$

$$\frac{\partial Error}{\partial out_1} = -(correct_1 - out_1)$$
$$= -(0.01 - 0.7513) = 0.7413$$

As $S'(x) = S(x) \cdot (1 - S(x))$

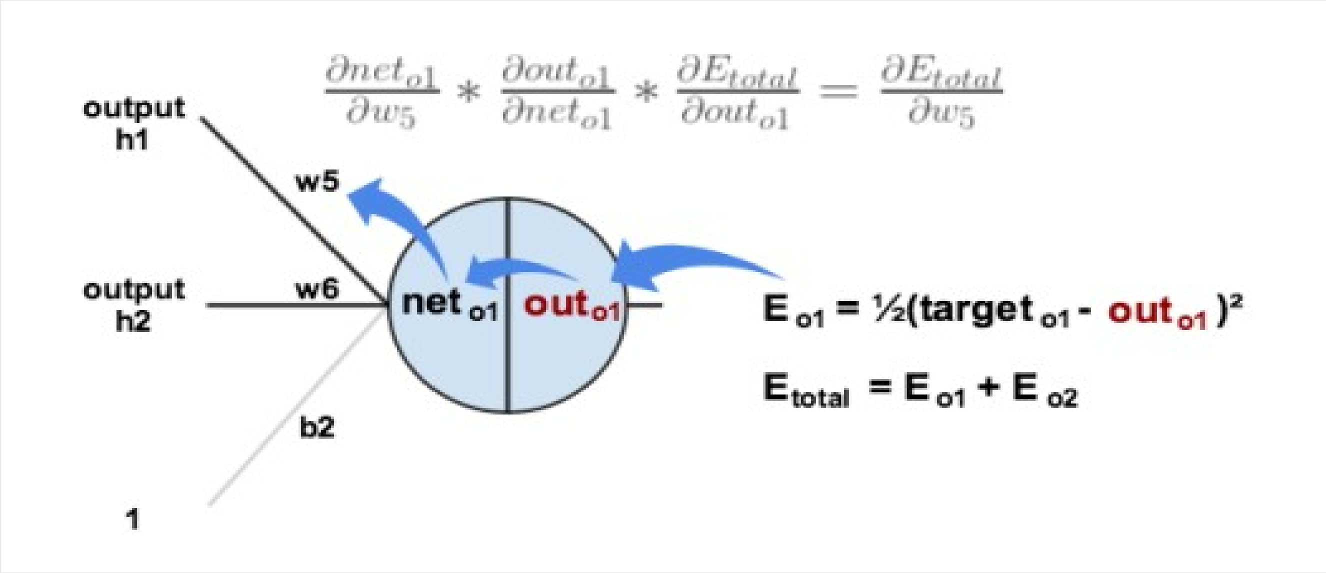
$$\frac{\partial S(In(N_3))}{\partial In(N_3)} = S(In(N_3)) \cdot (1 - S(In(N_3)))$$
$$= 0.7513 \cdot (1 - 0.7513) = 0.1868$$

$$\frac{\partial In(N_3)}{\partial w_5} = \frac{\partial w_5 \cdot Out(N_1) + w_6 \cdot Out(N_2) + b_2 \cdot 1}{\partial w_5}$$
$$= Out(N_1) = 0.5932$$

Thus, $\frac{\partial Error}{\partial w_5} = 0.7413 \cdot 0.1868 \cdot 0.5932 = 0.08217$

# Back-propagation

In a picture we did this:




Now that we know w5 is 0.08217 part responsible, we update the weight by:

$w_5 \leftarrow w_5 - \alpha * 0.08217 = 0.3589$ (from 0.4)

α is learning rate, set to 0.5

# Back-propagation

Updating this $w_5$ to $w_8$ gives:

$w_5 = 0.3589$

$w_6 = 0.4067$

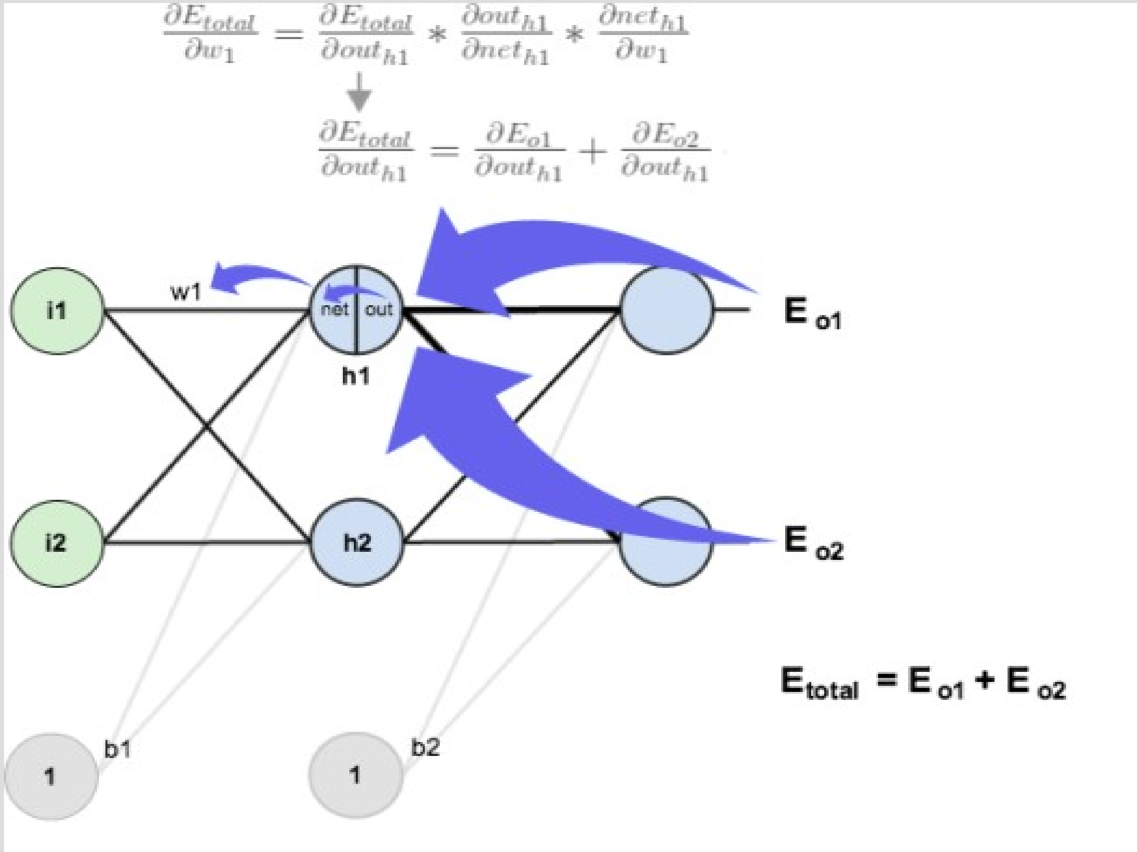
$w_7 = 0.5113$

$w_8 = 0.5614$

For other weights, you need to consider all possible ways in which they contribute

# Back-propagation

For $w_1$ it would look like:




(book describes how to dynamic program this)

# Back-propagation

Specifically for $w_1$ you would get:

$$\frac{\partial Error}{\partial S(In(N_1))} = \frac{\partial Error_1}{\partial S(In(N_1))} + \frac{\partial Error_2}{\partial S(In(N_1))}$$

$$\frac{\partial S(In(N_1))}{\partial In(N_1)} = S(In(N_1)) \cdot (1 - S(In(N_1)))$$
$$= 0.5933 \cdot (1 - 0.5933) = 0.2413$$

$$\frac{\partial In(N_3)}{\partial w_5} = \frac{\partial w_1 \cdot In_1 + w_2 \cdot In_2 + b_1 \cdot 1}{\partial w_5}$$
$$= In_1 = 0.05$$

Next we have to break down the top equation...

# Back-propagation

$$\frac{\partial Error}{\partial S(In(N_1))} = \frac{\partial Error_1}{\partial S(In(N_1))} + \frac{\partial Error_2}{\partial S(In(N_1))}$$

$$\frac{\partial Error_1}{\partial S(In(N_1))} = \frac{\partial Error_1}{\partial S(In(N_3))} \cdot \frac{\partial S(In(N_3))}{\partial In(N_3)} \cdot \frac{\partial In(N_3)}{\partial S(In(N_1))}$$

From before... $\frac{\partial Error_1}{\partial S(In(N_3))} \cdot \frac{\partial S(In(N_3))}{\partial In(N_3)}$
$= 0.7414 \cdot 0.1868 = 0.1385$

$$\frac{\partial In(N_3)}{\partial S(In(N_1))} = \frac{\partial w_5 \cdot S(In(N_1)) + w_6 \cdot S(In(N_2)) + b_1 \cdot 1}{\partial S(In(N_1))}$$
$$= w_5 = 0.4$$

Thus, $\frac{\partial Error_1}{\partial S(In(N_1))} = 0.1385 \cdot 0.4 = 0.05540$

# Back-propagation

Similarly for Error$_2$ we get:

$$\frac{\partial Error}{\partial S(In(N_1))} = \frac{\partial Error_1}{\partial S(In(N_1))} + \frac{\partial Error_2}{\partial S(In(N_1))}$$
$$= 0.05540 + -0.01905 = 0.03635$$

Thus, $\frac{\partial Error}{\partial w_1} = 0.03635 \cdot 0.2413 \cdot 0.05 = 0.0004386$

Update $w_1 \leftarrow w_1 - \alpha \frac{\partial Error}{\partial w_1} = 0.15 - 0.5 \cdot 0.0004386 = 0.1498$

You might notice this is small...
This is an issue with neural networks, deeper the network the less earlier nodes update